New Methods to Assess the Risk- Reward Profile of Non- Equity Products

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Mathematical Finance: The Founding Results

→ Harrison and Kreps (1979) revisit the key assumption of No Arbitrage in Black-Scholes (1973) and Merton (1973) and introduce the notion of *equivalent martingale measure* in a discrete-time setting (First Fundamental Theorem of Asset Pricing)

→ Harrison and Pliska (1981) extend the setting to continuous-time trading and exhibit the price of an attainable contingent claim as a *stochastic integral*

→ Harrison and Pliska (1983) examine the particular case of *complete* markets and establish the *unicity* of the equivalent martingale measure (Second Fundamental Theorem of Asset Pricing)

→ Delbaen-Schachermayer (1994, 1998) establish the First Fundamental Theorem for general semi- martingales (bounded or not)

→ Hence, all these results are essentially *model- independent*

Itô - Doeblin lemma: The History

→ Doeblin (1940)

→ Itô (1950)

→ And its consequences : the Partial Differential Equation satisfied by the call price and the *unicity* of this price as long as

. The underlying asset is continuously traded, hence visible and liquid

. And the underlying asset dynamics are represented by a uni-dimensional diffusion (for instance, a mean- reverting process in the case of commodities)

→ Extensions to stochastic volatility or jump processes: we may lose the market *completeness* in the Arrow - Debreu terminology, hence the uniqueness of the price , unless we have enough *primitive* (non- opaque and liquidly traded) to hedge the new sources of risk.

→ Obviously, the same issues were posed years ago in the case of credit derivatives, but ignored by the market players in a kind of "all win" game

Some Motivation for New Answers in Asset Price Modeling and Risk Measuring

→ From a financial viewpoint, normality of returns was/is a key issue

→ in the mean-variance paradigm

- Markowitz frontier
- Capital Asset Pricing Model and Sharpe Ratio

→ in the Black-Scholes-Merton formula as well as Black formula, Garman-Kohlhagen….

→ In Value at Risk or a better risk measure of maximum potential loss at a given horizon with a given probability: importance of tails!

Empirical Evidence for Non-Normality

- Skewness
- Kurtosis

The Dynamics of Assets Returns

→ Fama (early 60's) exhibits that the deviations from normality increase when the time horizon over which the returns are measured decreases (in particular, long tails)

→ Mandelbrot (1967) proposes stable processes as a better representation of price processes. His ideas continue to be followed today by some researchers

→ Clark (1973) studies cotton Futures prices and rejects Mandelbrot's proposal; instead, he writes the stock price process as a *subordinated* process *with finite moments*

$$\ln S(t) = Y(X(t)) \qquad (1)$$

→ where

- Y is assumed to be Brownian motion
- the subordinator X(t) is supposed to have log-normally distributed increments and to represent the cumulative volume

Transaction Clock and Normality of Returns

→ G-Ané (*RISK*, 1996) and Ané- G (*JOF*, 2000) legitimate and extend Clark's remarkable results through the two key steps :

→ By the first fundamental theorem of asset pricing, No arbitrage implies that asset prices are semimartingales under the physical measure P

→ Monroe's theorem (1978) establishes that "any semimartingale is a time-changed Brownian motion"

Hence the (log) price may be written

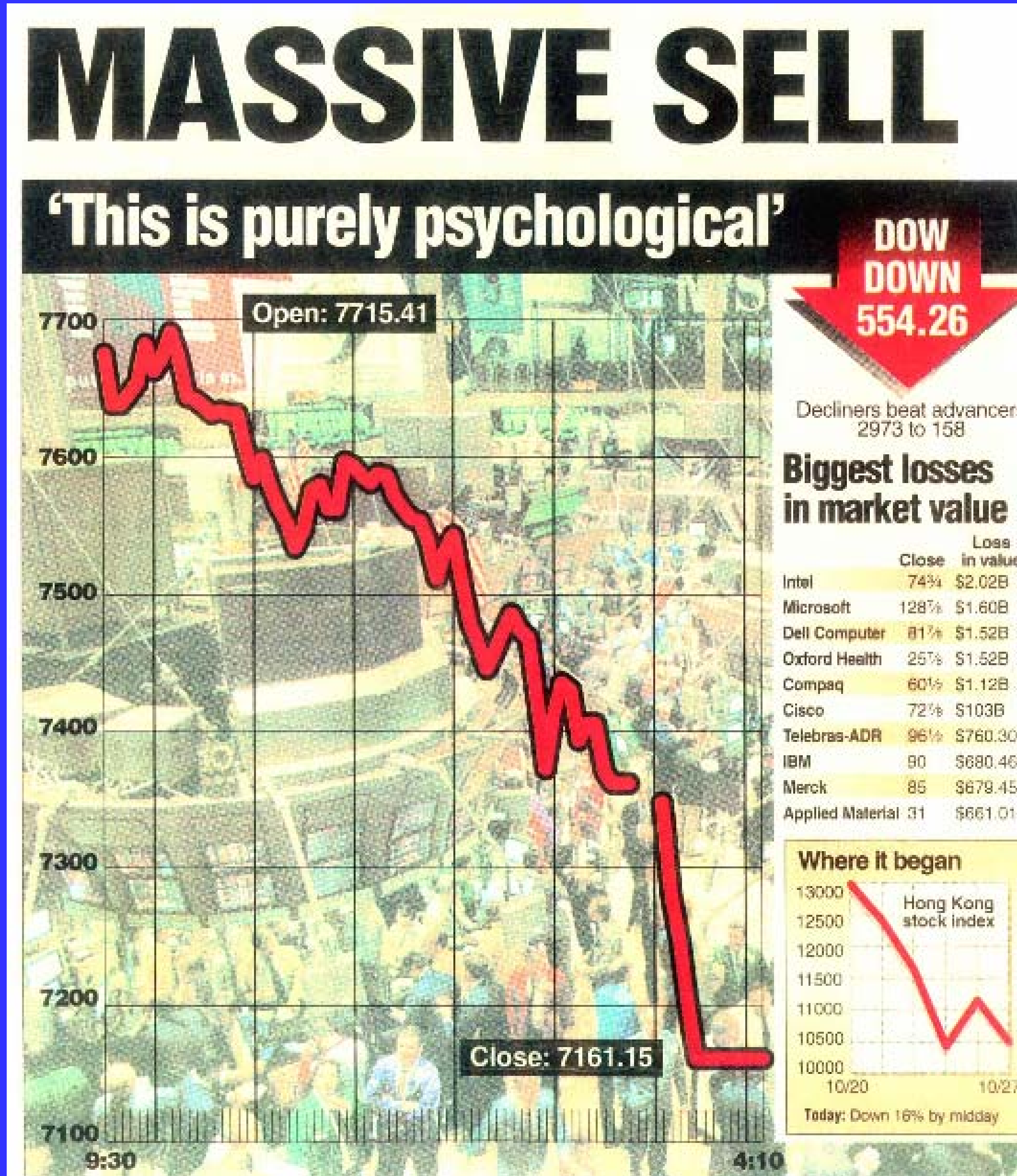
$$\ln S(t) = W(T(t))$$

where T(t) is an almost increasing process (and is likely *not* to be a subordinator)

→ Derman (2001) argues that in times of speculative excitement, investors may perceive and experience the risk and return of an asset in "intrinsic time" and defines the *temperature* of a stock in the following way

$$\text{asset temperature} = \text{asset traditional volatility} \bullet \sqrt{\textit{tradingfrequency}}$$

He states that in a one-factor market (Capital Asset Pricing Model type) and for short-term speculators, the benchmark return should be equal to beta times the market return, enhanced by a factor equal to the square root of the ration of the trading frequency of the stock to that of the market



MASSIVE SELL

'This is purely psychological'

DOW DOWN 554.26

Open: 7715.41

Close: 7161.15

7700
7600
7500
7400
7300
7200
7100
9:30
4:10

Decliners beat advancers 2973 to 158

Biggest losses in market value

	Close	Loss in value
Intel	74¾	$2.02B
Microsoft	128⅞	$1.60B
Dell Computer	81⅞	$1.52B
Oxford Health	25⅞	$1.52B
Compaq	60½	$1.12B
Cisco	72⅞	$103B
Telebras-ADR	96½	$760.30
IBM	90	$680.46
Merck	85	$679.45
Applied Material	31	$661.01

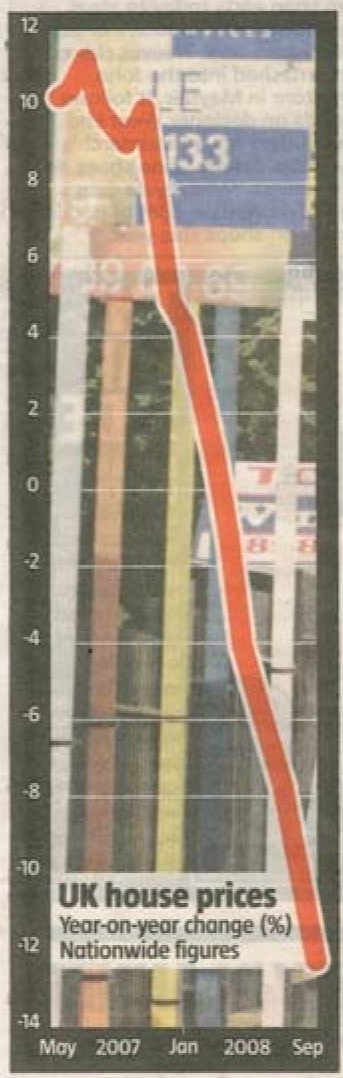
Where it began

Hong Kong stock index

13000
12500
12000
11500
11000
10500
10000
10/20
10/27

Today: Down 16% by midday

NATIONWIDE REPORTS RECORD 12.4% DROP IN A YEAR



Property prices fall off a cliff

HOUSE prices in London have plummeted a record amount in the past year, new figures confirm today.

The unprecedented 9.4 per cent drop has wiped almost £30,000 off the price of a house and is a direct result of the City financial crisis, according to mortgage lender Nationwide. The

BY JONATHAN PRYNN AND HUGO DUNCAN

average price of a property in the capital now stands at £274,124, down from £302,486.

Further steep falls are expected in the rest of the year and nationally prices were down 12.4 per cent. The revelation puts even more pressure on the Bank of England to act and raises the likelihood of an interest rate cut next week.

The figures also reveal that one of Britain's best-known builders is slashing its asking price for new homes by almost half, as property values endure their biggest slump in value. Barratt Developments has been forced to make

Continued on Page 4▶

The Case for Discontinuous Processes : Empirical Observations

→ From Time Series Data

- It has been known since early work by Fama that daily returns are more long-tailed relative to the normal density and that they approach normality when we consider monthly returns
- No have got available in the last few years high-frequency data and the returns over time intervals of one hour, 15 minutes, 1 minute, confirm and exaggerate this property
- Looking for instance at S&P 500 returns, the kurtosis goes to

  13.85 for 15-minute returns

  8.59 for 1-minute returns
- Distributions are more peaked than the normal distribution
- Having the same volatility under the real measure P and the risk-adjusted probability measure Q is a very hard constraint imposed by the geometric Brownian motion assumption

The Case for Pure Jump Processes
(G-Madan-Yor : 2000, 2001)

→ Index and stock prices are moving by jumps

→ Pure jump processes are of finite variation, as are real price processes

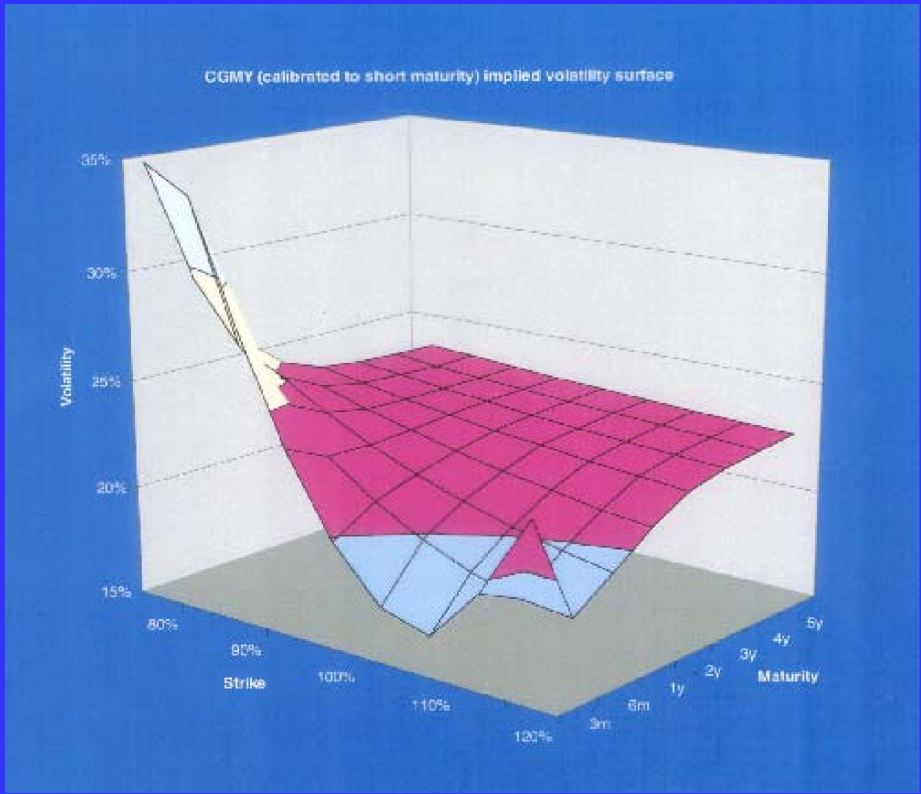
→ Writing ln S(t) = W(T(t)) it is clear that the continuity of the process (S(t)) is equivalent to the continuity of the process T(t)

If T(t) is continuous, then

$$T(t) = \int_0^t a(u)\,du + \int_0^t b(u)\,dZ(u)$$

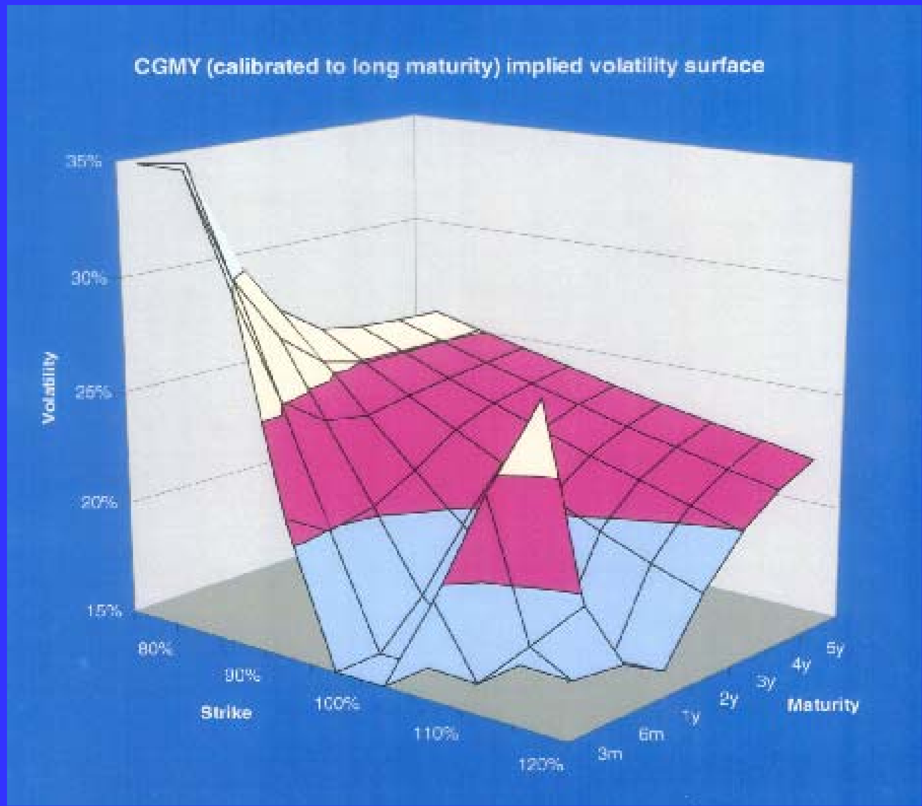
Because T(t) is increasing, b(u) ≡ 0 and the time change is locally deterministic. This is an undesirable property since that we believe T(t) is related to locally random market activity like the arrival of orders or information

CGMY volatility surface : short calibration



- Very flat at long maturity

CGMY volatility surface : long calibration



- Much better at long maturity
- Short maturity bad

Introducing Stochastic Volatility in Lévy processes (Math Finance, 2004)

→ Homogeneous Lévy process fit statistical data quite well but impose too strict conditions on the term structure of risk–neutral variances : in particular, the variance rate is constant across maturities

→ Heston (1993), Bates (1996), Barndorff-Nielsen and Shephard (2001) show that volatilities estimated from option time series are stochastic and usually clustered

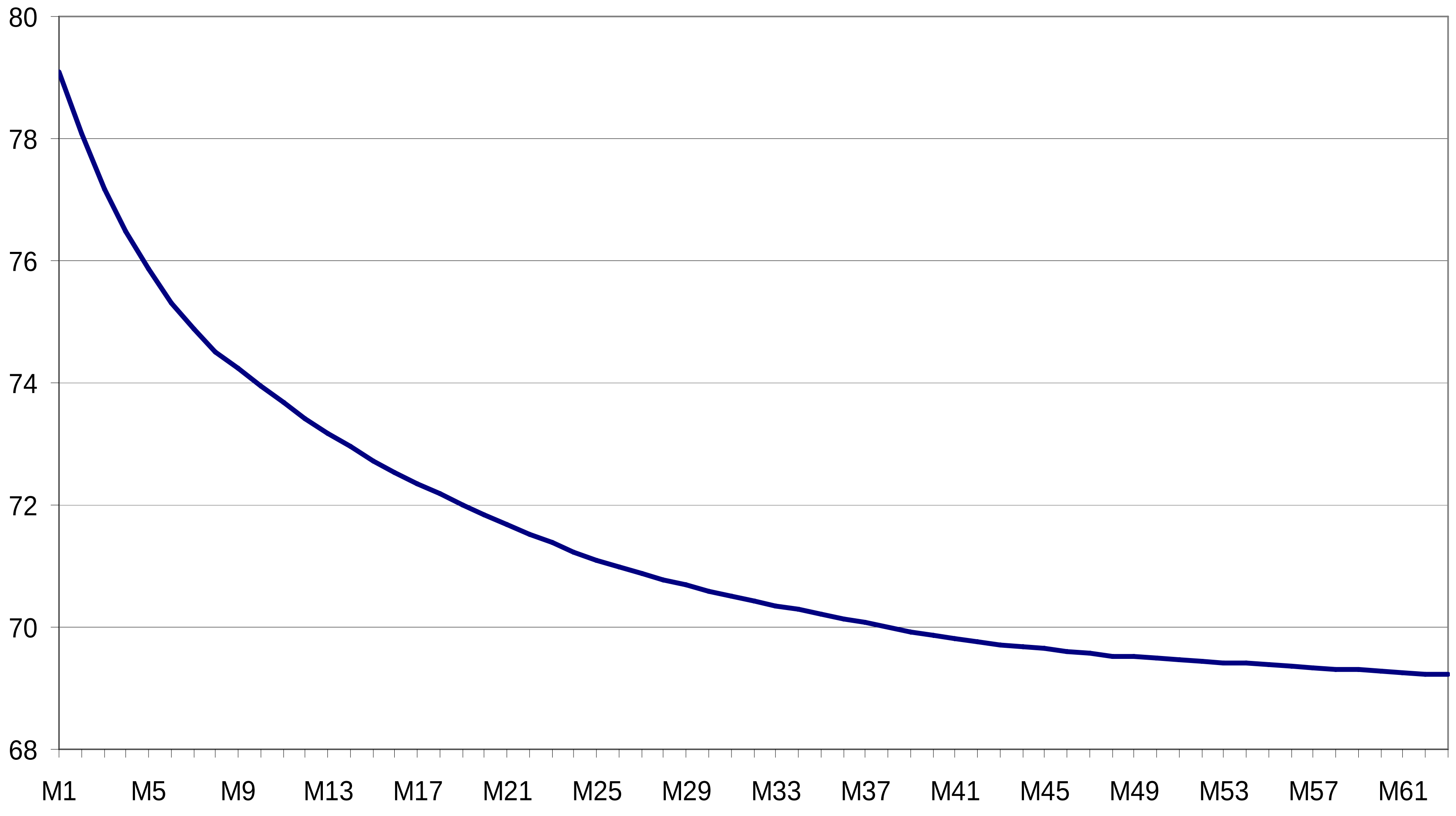
→ Random changes in volatility can be produced by random changes in time, as first proposed by G- Yor (1993)

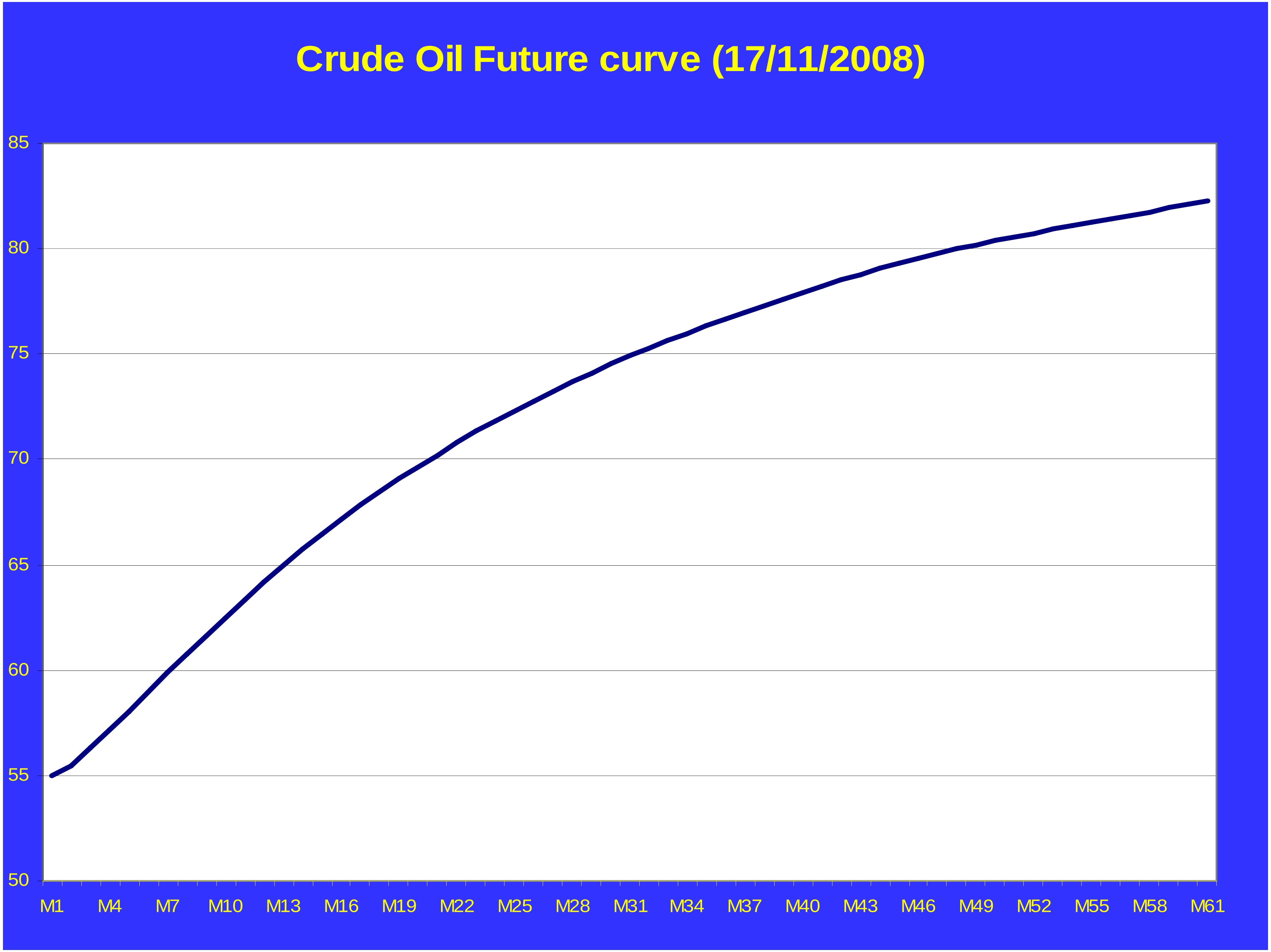
→ The rate of time change must be positive for the transaction clock to be increasing

→ This rate should be reverting in order for the random time change (and volatility) to persist

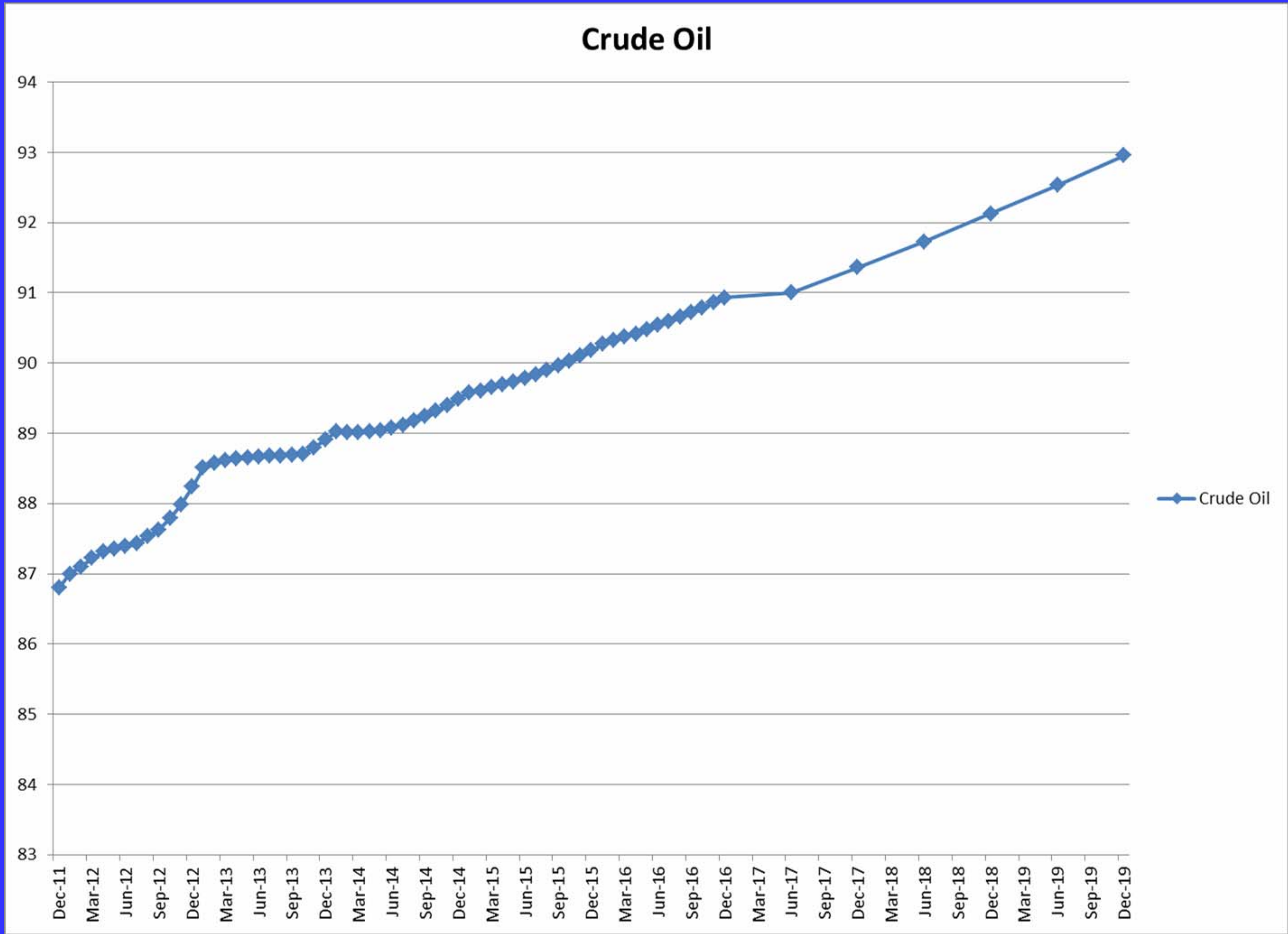
WTI Oil Prices Jan 2002 - Oct 2007

‘Normal’ Backwardation in September 2007





Crude Oil Future curve (17/11/2008)
85
80
75
70
65
60
55
50
M1
M4
M7
M10
M13
M16
M19
M22
M25
M28
M31
M34
M37
M40
M43
M46
M49
M52
M55
M58
M61



Crude Oil
94
93
92
91
90
89
88
87
86
85
84
83
Dec-11
Mar-12
Jun-12
Sep-12
Dec-12
Mar-13
Jun-13
Sep-13
Dec-13
Mar-14
Jun-14
Sep-14
Dec-14
Mar-15
Jun-15
Sep-15
Dec-15
Mar-16
Jun-16
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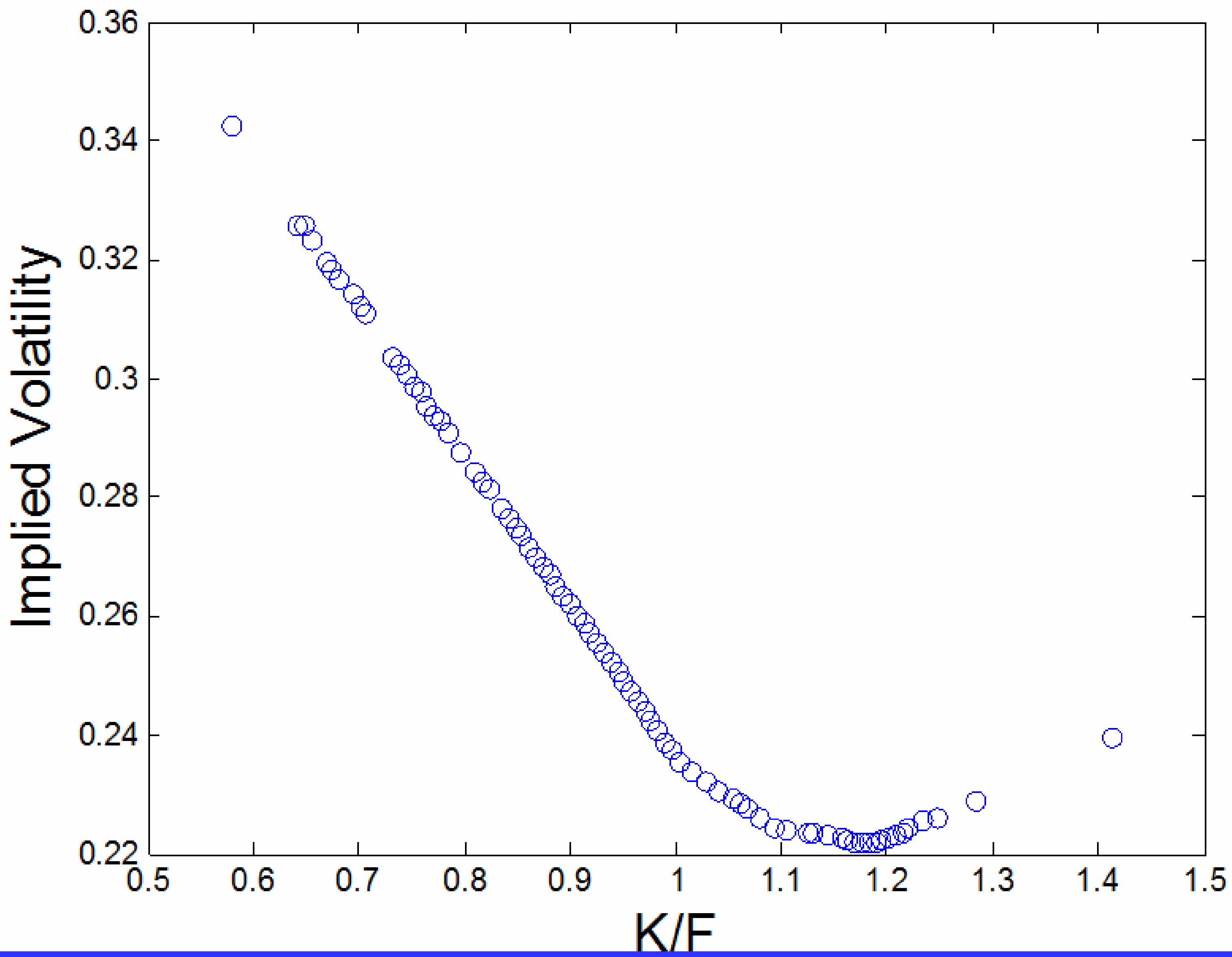
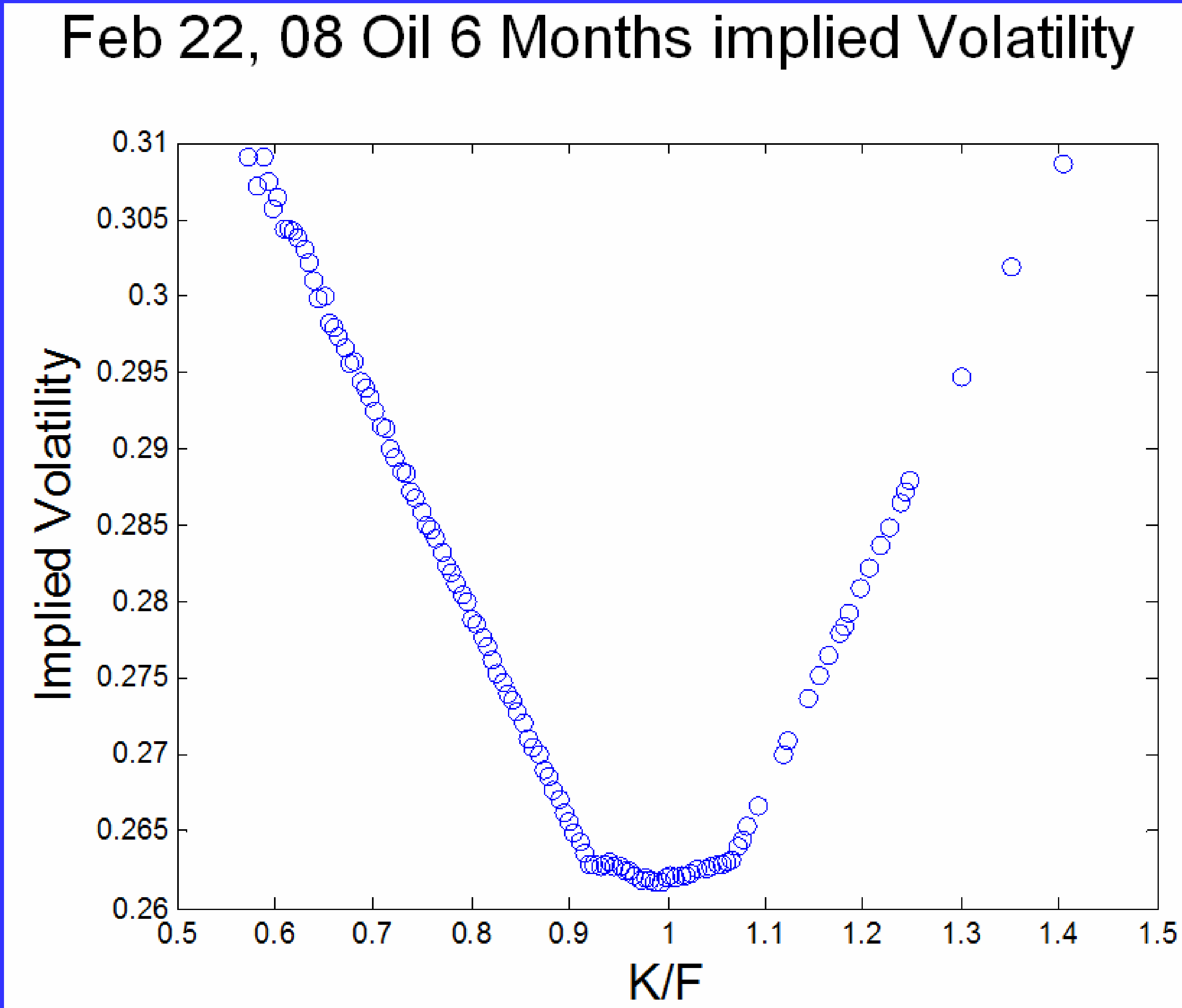
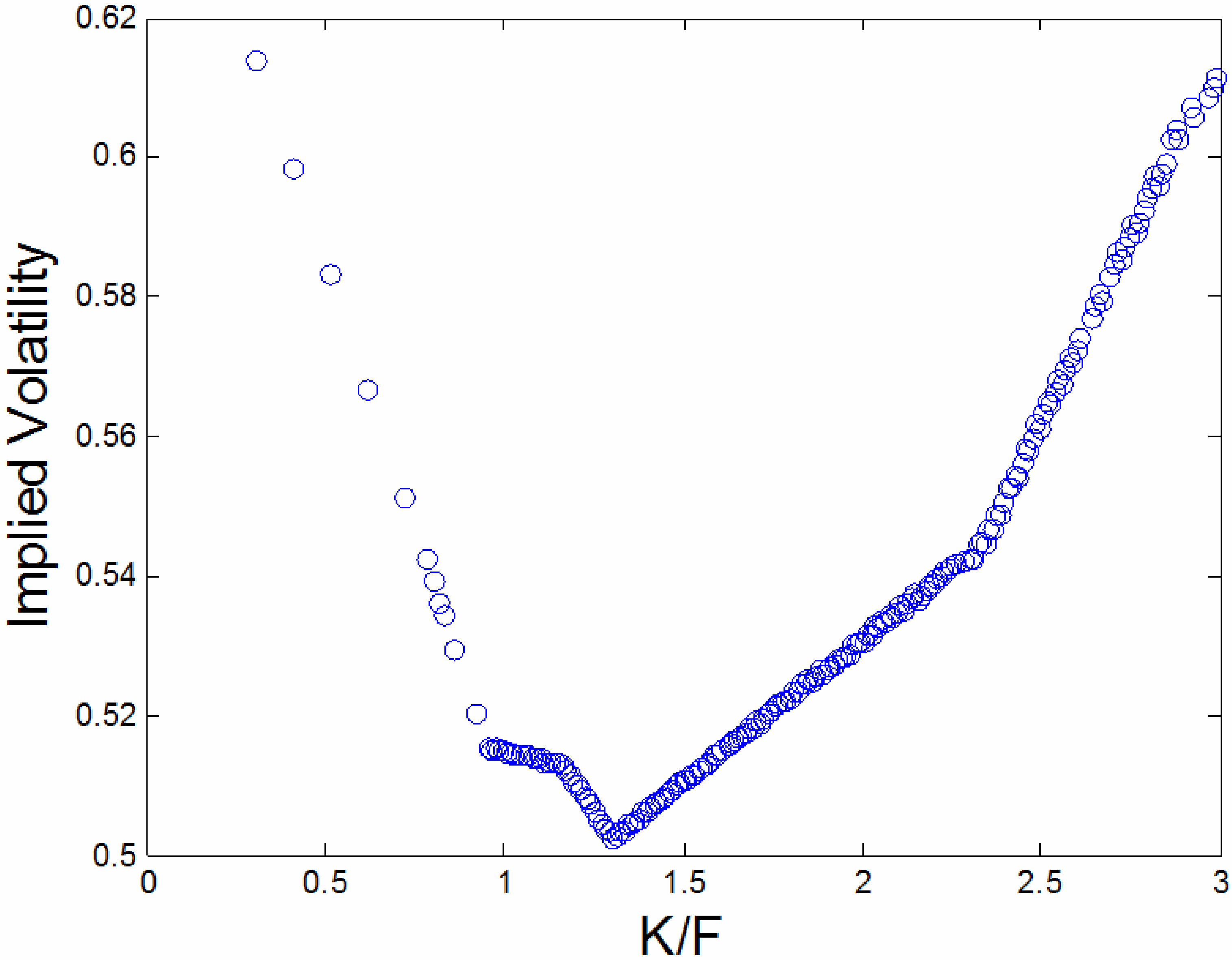
Mar-17
Jun-17
Sep-17
Dec-17
Mar-18
Jun-18
Sep-18
Dec-18
Mar-19
Jun-19
Sep-19
Dec-19
Crude Oil

Sep 21, 07 Oil 6 Months implied Volatility

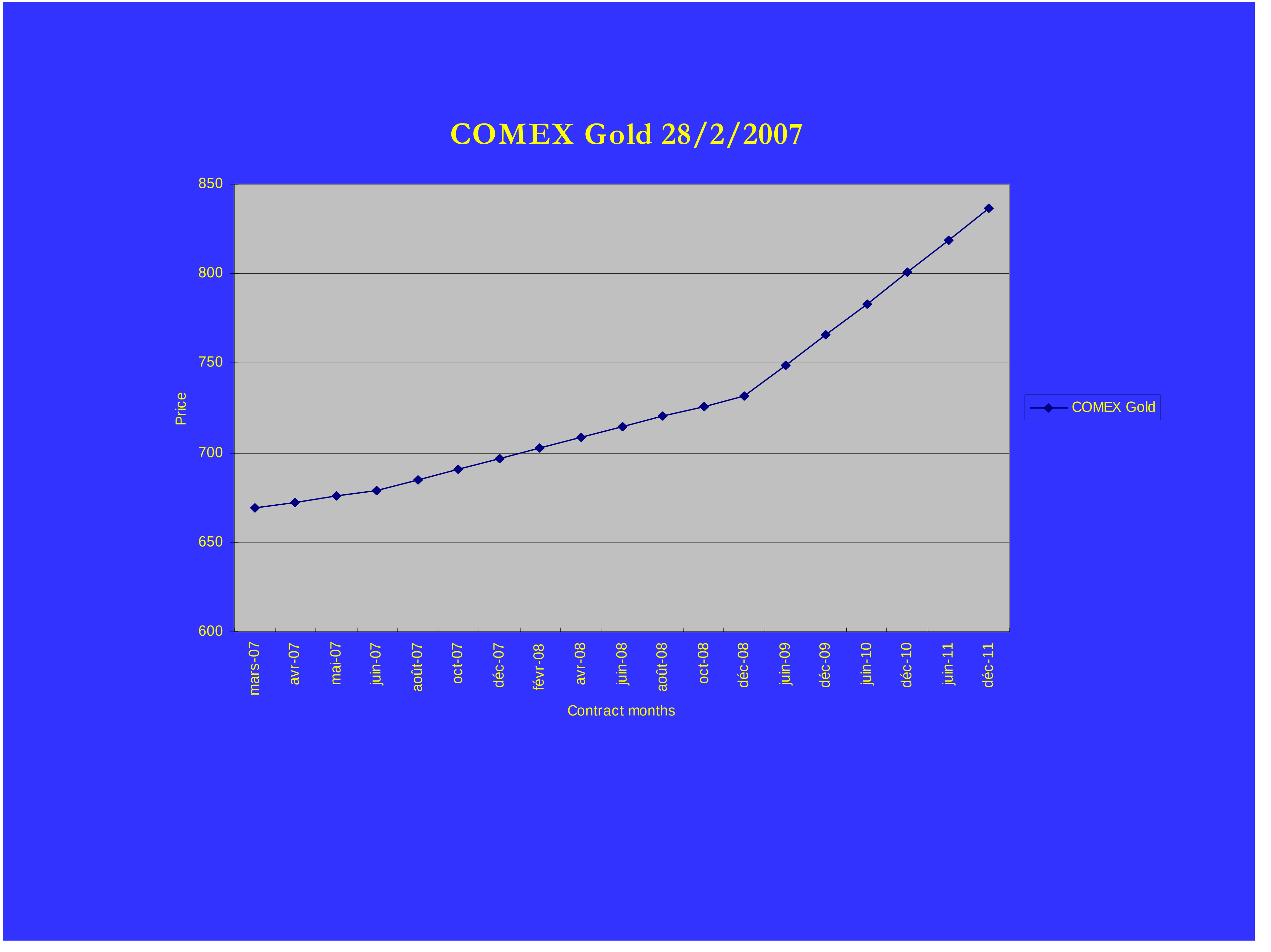



Feb 22, 08 Oil 6 Months implied Volatility
Implied Volatility
K/F
0.31
0.305
0.3
0.295
0.29
0.285
0.28
0.275
0.27
0.265
0.26
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5

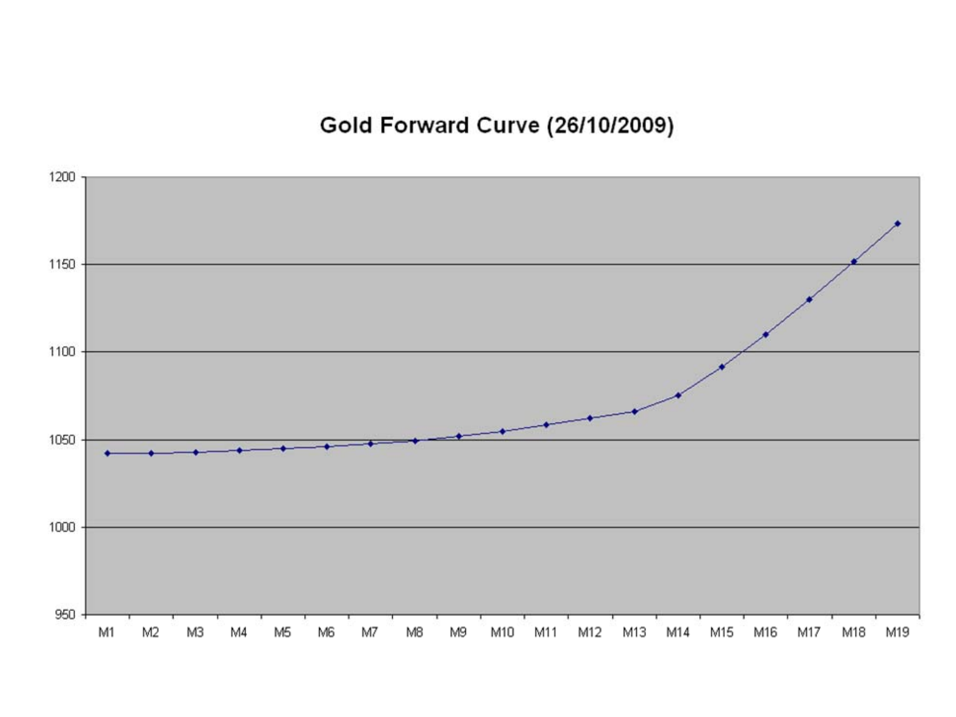
Feb 23, 09 Oil 6 Months implied Volatility



COMEX Gold Prices - 2002 to 2010

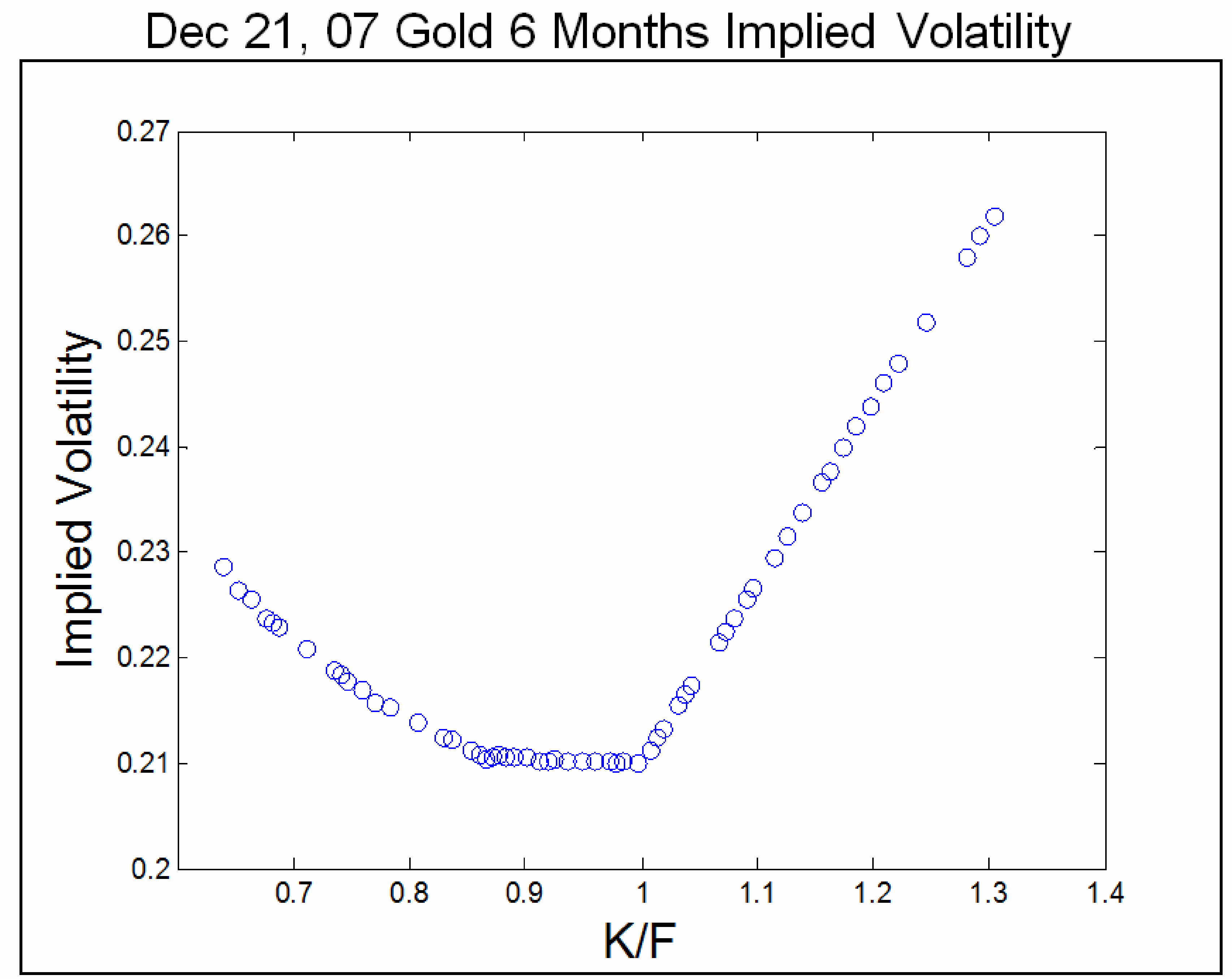

COMEX Gold 28/2/2007
850
800
750
700
650
600
Price
mars-07
avr-07
mai-07
juin-07
août-07
oct-07
déc-07
févr-08
avr-08
juin-08
août-08
oct-08
déc-08
juin-09
déc-09
juin-10
déc-10
juin-11
déc-11
Contract months
COMEX Gold

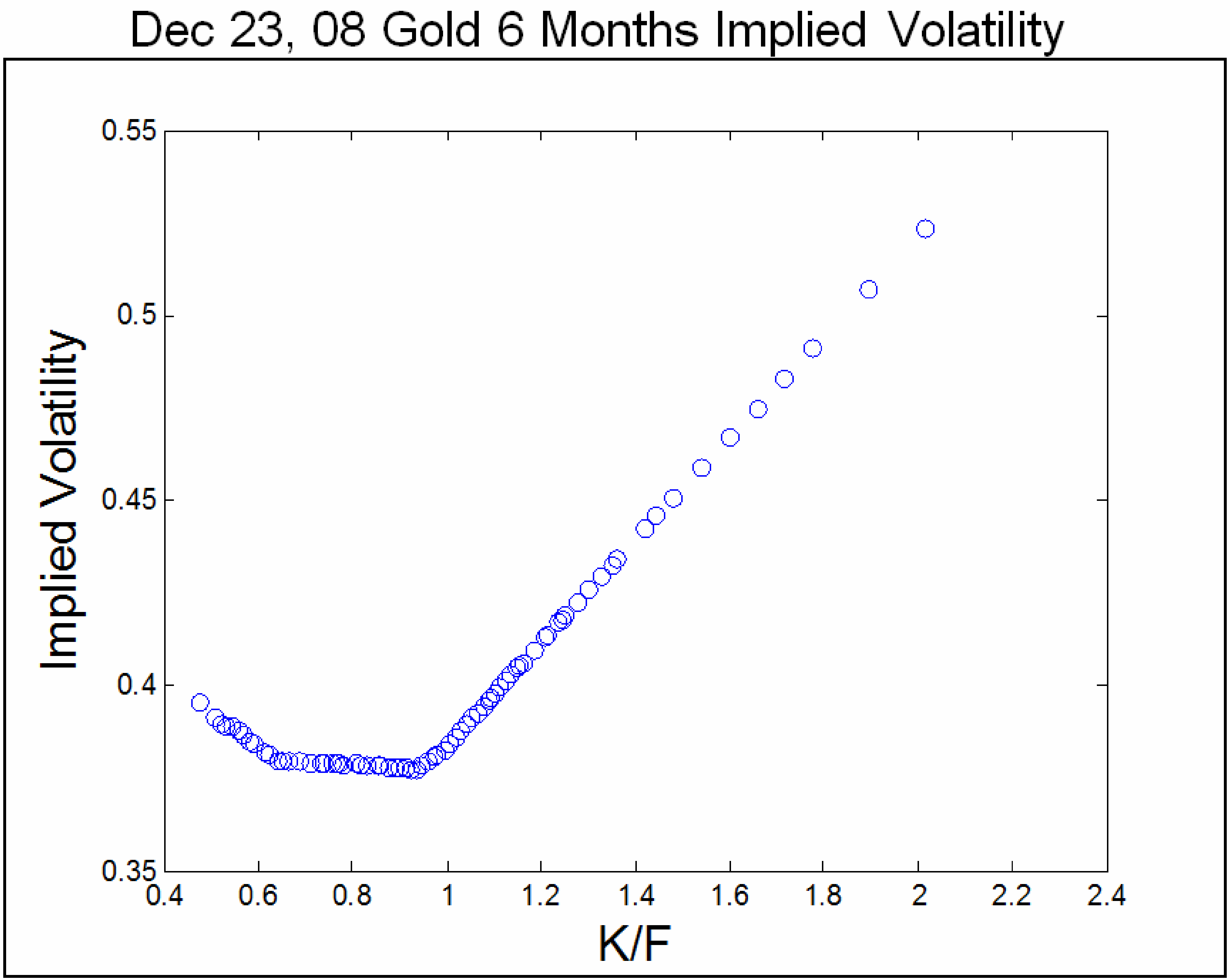
Gold Forward Curve, October 26, 2009



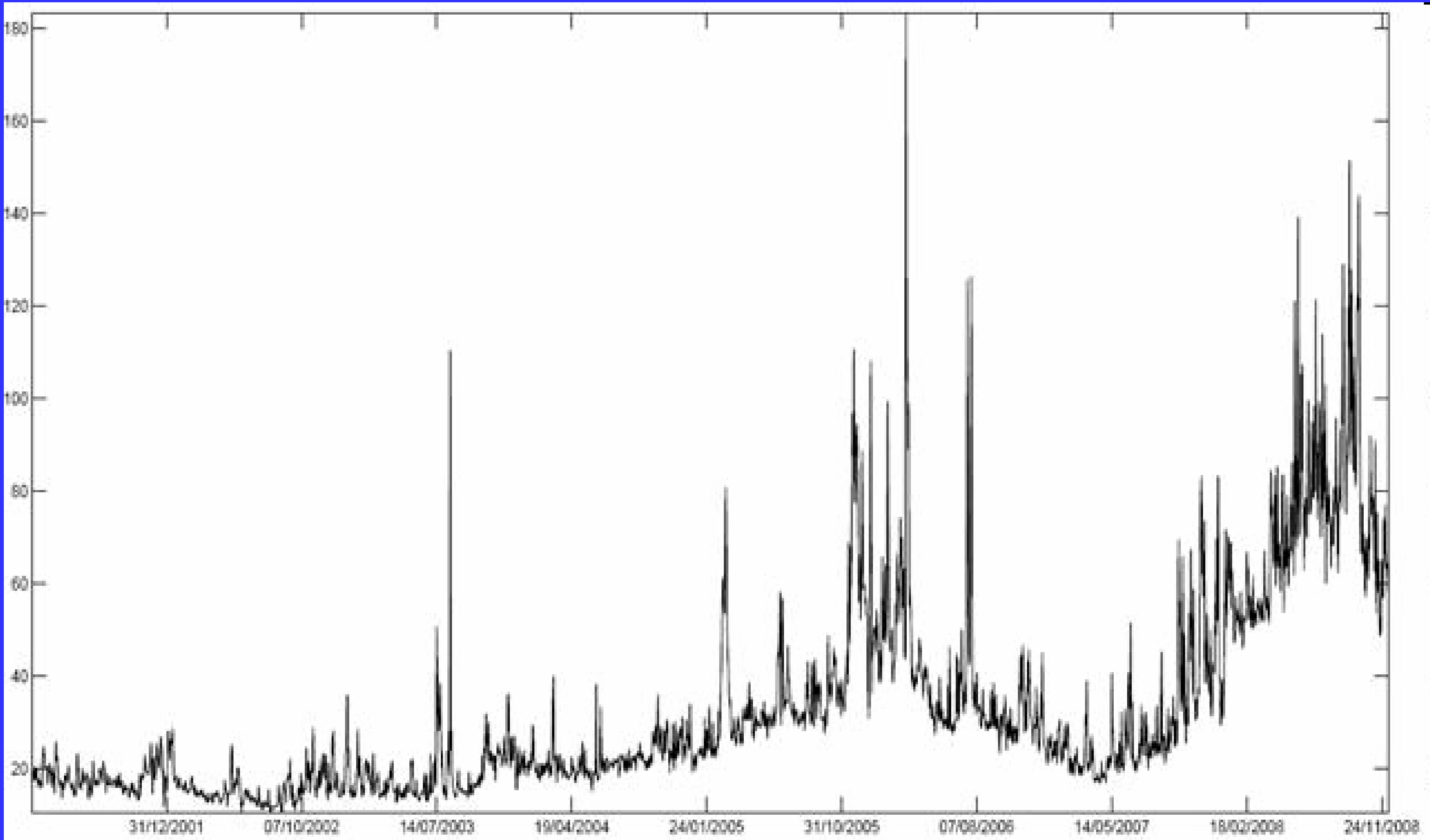
Number of Ounces of Gold that can buy the Average US House

Chart created with NeoTicker EOD © 1998-2009 TickQuest Inc.

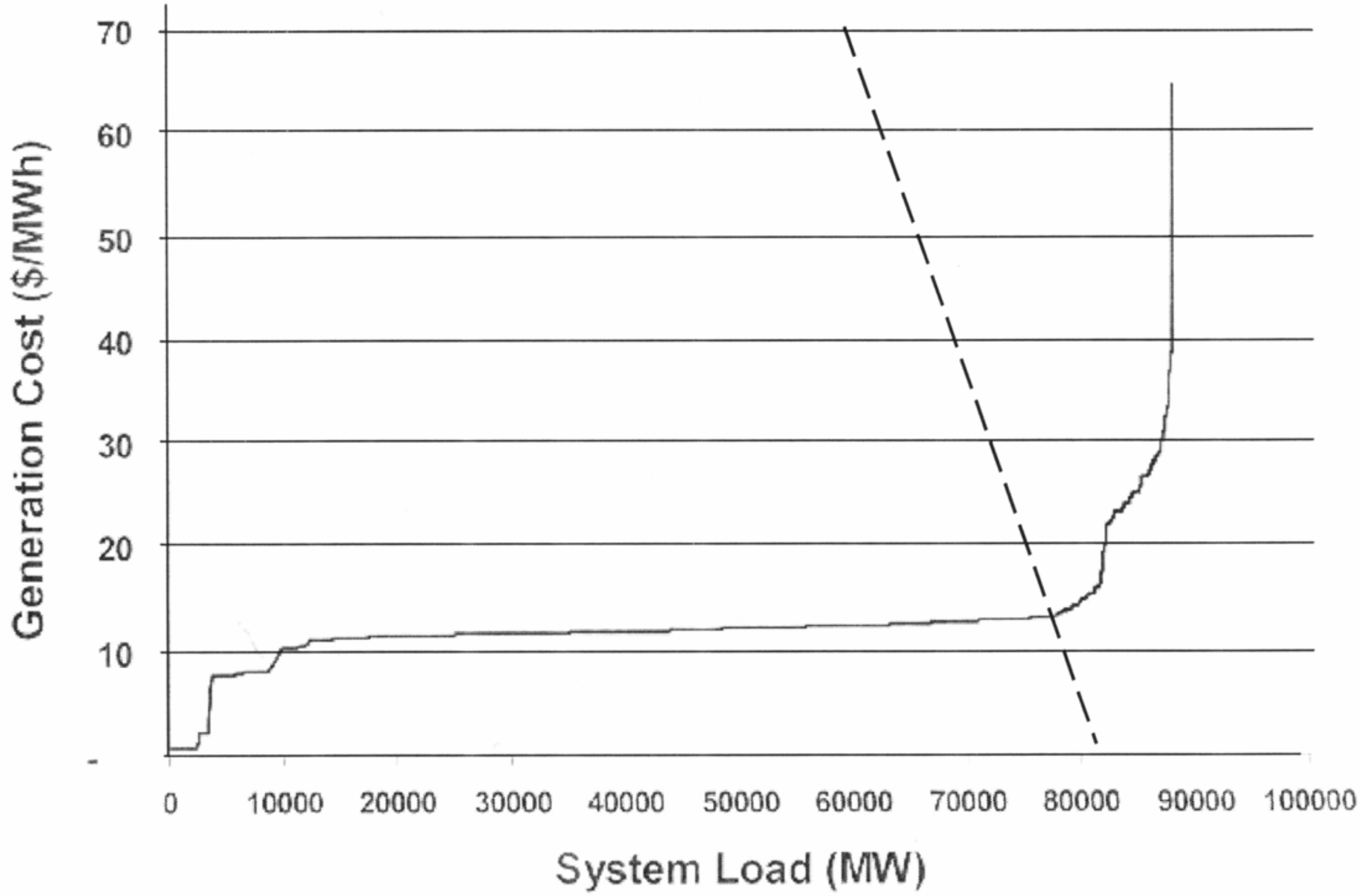

Dec 21, 07 Gold 6 Months Implied Volatility
Implied Volatility
K/F
0.27
0.26
0.25
0.24
0.23
0.22
0.21
0.2
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4


Dec 23, 08 Gold 6 Months Implied Volatility
Implied Volatility
0.55
0.5
0.45
0.4
0.35
0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2 2.2 2.4
K/F

An Example of Electricity Price Trajectory



ECAR GENERATION CURVE



Baltic Dry Index – 2000 to 2011

Checking that a Model for Spot Prices is Acceptable

→ Realize that going from $4500 to $70 is definitely a jump downward. Hence, the model should allow for positive and negative jumps

→ Generate with the model a variety of trajectories and check that a least some of them look like real trajectories

Trajectorial Adequacy of the model

→ Compute the first 4 moments of the calibrated model and of the real trajectory and verify that they are similar

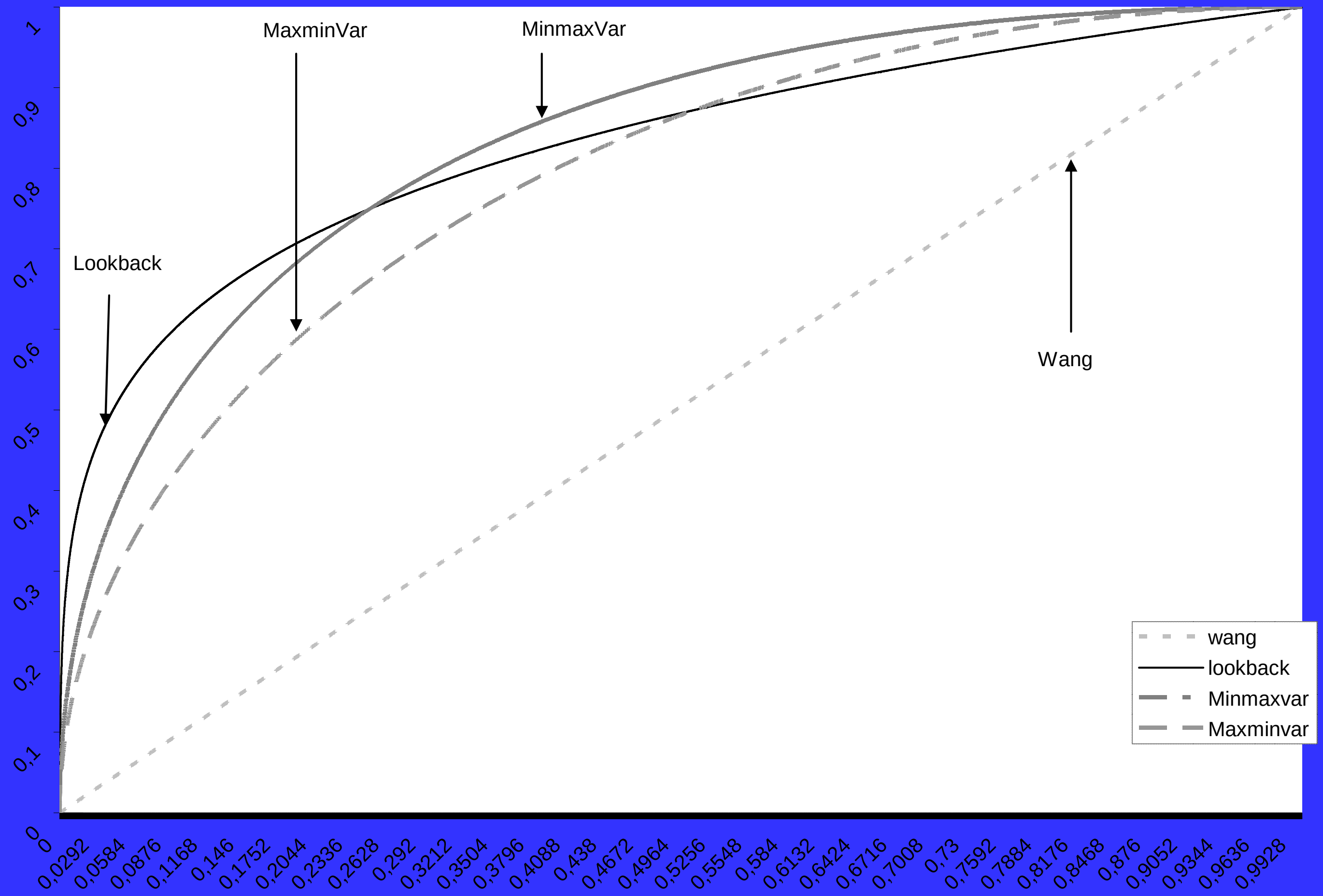
Statistical Adequacy of the model

→ For instance, introducing only upward jumps generates a very highly positive skewness which is not observed in practice

Distortions Risk Measures in Probability

→ A probability space (Ω, F, P) is considered where Ω represents the set of states of nature; F is the filtration of information and P the real probability measure. Hedge fund returns or Net Asset Value (NAV) final values are random variables X defined on (Ω, F, P). X is characterised by its cumulative distribution function F classically defined by $F(x) = P\left[X \leq x\right]$. A function $g_\alpha(.)$ is introduced, defining a mapping of the interval [0;1] on itself, that is strictly increasing and continuous; α, a strictly positive number is the parameter of the distortion function defined as follows $F \xrightarrow[g_\alpha]{} F^*$ where for any $x \in \Re$, $F^*(x) = g_\alpha\left(F(x)\right)$.

Distortion Functions for Hedge Funds (G. Kharoubi, 2011)



Distortion Risk Measure for Hedge Funds: Some Examples

Comparison between VaR, ETL and lookback DRM for a $ 1 million position

alpha= 0,1

	Distressed	Macro	Event-Driven	Relative value	Market neutral	S&P 500
Lookback DRM	36976	33081	27910	32982	27910	83218
EVaR	39748	30117	26862	33773	26865	74640
VaR	2157	5150	3407	2451	3407	12697

alpha= 0,05

	Distressed	Macro	Event-Driven	Relative value	Market neutral	S&P 500
Lookback DRM	40709	35440	30182	35723	30182	88234
EVaR	38715	28281	25206	32042	25208	68610
VaR	3358	7369	5332	4391	5332	19684

alpha= 0,01

	Distressed	Macro	Event-Driven	Relative value	Market neutral	S&P 500
Lookback DRM	42306	36415	31136	36869	31136	90252
EVaR	33842	21410	20345	25483	20347	43930
VaR	8464	14684	10496	11237	10496	45559

Hélyette Geman

Hélyette GEMAN is a Professor of Finance at Birkbeck, University of London and ESCP Europe. She is a graduate of Ecole Normale Superieure in mathematics, holds a Masters degree in theoretical physics and a PhD in mathematics from the University Pierre et Marie Curie and a PhD in Finance from the University Pantheon Sorbonne. Professor Geman has been a scientific advisor to a number of major energy companies for the last decade, covering the spectrum of oil, natural gas and electricity as well as agricultural commodities origination and trading ; and was previously the head of Research and Development at Caisse des Depots. She has published more than 120 papers in major finance journals including the Journal of Finance, Mathematical Finance, Journal of Financial Economics, Journal of Banking and Finance and Journal of Business. She has also written a book entitled *Insurance and Weather Derivatives* and is a Member of Honor of the French Society of Actuaries. Professor Geman's research includes asset price modelling using jump-diffusions and Lévy processes, commodity forward curve modelling and exotic option pricing for which she won the first prize of the Merrill Lynch Awards. She was the first President of the Bachelier Finance Society and was named in 2004 in the Hall of Fame of Energy Risk. Her book *Commodities and Commodity Derivatives,* published by Wiley Finance in 2005, has become the reference in the field.

Professor Geman is a Member of the Board of the UBS-Bloomberg Commodity Index and a Scientific Advisor to the European Union on Agricultural Commodities