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Agent based modeling of financial markets

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Brian Heath, Raymond Hill and Frank Ciarallo (2009)

A Survey of Agent-Based Modeling Practices (January 1998 to July 2008)

Journal of Artificial Societies and Social Simulation **12** (4) 9

“Emerging from the fields of Complexity, Chaos, Cybernetics, Cellular Automata and Computer Science, the Agent-Based Modeling (ABM) simulation paradigm began popularity in the 1990s and represent a departure from the more classical simulation approaches such as the discrete-event simulation paradigm. ..... This means the ABM paradigm can represent large systems consisting of many subsystem interactions. These systems are typically characterized as being unpredictable, decentralized and nearly decomposable”

Some examples

Brown, D. G., and D. T. Robinson 2006. Effects of heterogeneity in residential preferences on an agent-based model of urban sprawl. *Ecology and Society* **XX**(YY): ZZ. [online] URL: http://www.ecologyandsociety.org/volXX/issYY/artZZ/

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Research, part of a Special Feature on Empirical based agent-based modeling

Effects of Heterogeneity in Residential Preferences on an Agent-Based Model of Urban Sprawl

Daniel G. Brown[1] *and Derek T. Robinson*[1]

ABSTRACT. The ability of agent-based models (ABMs) to represent heterogeneity in the characteristics and behaviors of actors enables analyses about the implications of this heterogeneity for system behavior. The importance of heterogeneity in the specification of ABMs, however, creates new demands for empirical support. An earlier analysis of a survey of residential preferences within southeastern Michigan revealed seven groups of residents with similar preferences on similar characteristics of location. In this paper, we present an ABM that represents the process of residential development within an urban system and run it for a hypothetical pattern of environmental variation. Residential locations are selected by residential agents, who evaluate locations on the basis of preference for nearness to urban services, including jobs, aesthetic quality of the landscape, and their similarity to their neighbors. We populate our ABM with a population of residential preferences drawn from the survey results in five different ways: (1) preferences drawn at random; (2) equal preferences based on the mean from the entire survey sample; (3) preferences drawn from a single distribution, whose mean and standard deviation are derived from the survey sample; (4) equal preferences within each of seven groups, based on the group means; and (5) preferences drawn from distributions for each of seven groups, defined by group means and standard deviations. Model sensitivity analysis, based on multiple runs of our model under each case, revealed that adding heterogeneity to agents has a significant effect on model outcomes, measured by aggregate patterns of development sprawl and clustering.

Key Words: *complex systems; social surveys; spatial modeling; urban sprawl*.

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BioWar: Scalable Agent-Based Model of Bioattacks

Kathleen M. Carley, Douglas B. Fridsma, Elizabeth Casman, Alex Yahja, Neal Altman,
Li-Chiou Chen, Boris Kaminsky, and Démian Nave

***Abstract*—While structured by social and institutional networks, disease outbreaks are modulated by physical, economical, technological, communication, health, and governmental infrastructures. To systematically reason about the nature of outbreaks, the potential outcomes of media, prophylaxis, and vaccination campaigns, and the relative value of various early warning devices, social context, and infrastructure, must be considered. Numerical models provide a cost-effective ethical system for reasoning about such events. BioWar, a scalable citywide multiagent network numerical model, is described in this paper. BioWar simulates individuals as agents who are embedded in social, health, and professional networks and tracks the incidence of background and maliciously introduced diseases. In addition to epidemiology, BioWar simulates health-care-seeking behaviors, absenteeism patterns, and pharmaceutical purchases, information useful for syndromic and behavioral surveillance algorithms.**

***Index Terms*—Bioterrorism, multiagent network, social network, syndromic and behavioral surveillance.**

illustrated the importance of and close linkage among social networks [1], [2], disease transmission, and early detection. Like natural epidemics, biological attacks will also unfold within spatially defined complex social systems, and the societal response will have profound effects on their outcome.

It is not always clear how to best detect and respond to a disease outbreak, either natural or malicious. The goal of our research is to develop tools to simulate how diseases spread through socially connected groups so that these tools may be used to test the various detection and response options. This paper focuses on bioterrorist attacks, but the model structure has been applied to emergent and familiar diseases as well.

In trying to prepare for attacks, policy makers need to be able to think through the consequences of their decisions in various situations. Role-playing "simulation" physical exercises can provide valuable insights, but they are limited in number,

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Identifying control mechanisms of granuloma formation during *M. tuberculosis* infection using an agent-based model

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Available online 26 August 2004

Abstract

Infection with *Mycobacterium tuberculosis* is a major world health problem. An estimated 2 billion people are presently infected and the disease causes approximately 3 million deaths per year. After bacteria are inhaled into the lung, a complex immune response is triggered leading to the formation of multicellular structures termed granulomas. It is believed that the collection of host granulomas either contain bacteria resulting in a latent infection or are unable to do so, leading to active disease. Thus, understanding granuloma formation and function is essential for improving both diagnosis and treatment of tuberculosis. Granuloma formation is a complex spatio–temporal system involving interactions of bacteria, specific immune cells, including macrophages, CD4+ and CD8+ T cells, as well as immune effectors such as chemokine and cytokines. To study this complex dynamical system we have developed an agent-based model of granuloma formation in the lung. This model combines continuous representations of chemokines with discrete agent representations of macrophages and T cells in a cellular automata-like environment. Our results indicate that key host elements involved in granuloma formation are chemokine diffusion, prevention of macrophage overcrowding within the granuloma, arrival time, location and number of T cells within the granuloma, and an overall host ability to activate macrophages. Interestingly, a key bacterial factor is its intracellular growth rate, whereby slow growth actually facilitates survival.

Keywords: Granuloma formation; Tuberculosis; Agent based model; *Mycobacterium tuberculosis*

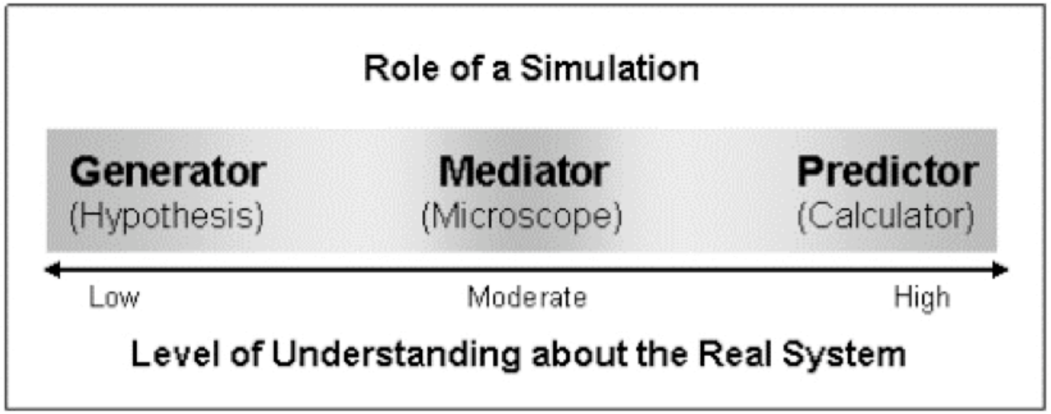

Figure 4. Purpose of the Simulation

- A Generator is a simulation where little is known about the system of interest and it is used primarily to determine if a given conceptual model/theory is capable of generating observed behavior of the system.
- A Mediator is a simulation where the system is moderately understood and it is used primarily to establish the capability of the conceptual model to represent the system and to then gain some insight into the system's characteristics and behaviors.
- A Predictor is a simulation where the system is well understood and it is used primarily to estimate or predict a system's behavior with little time spent on ensuring that the conceptual model is correct because this aspect of the simulation has already been established.

KUPPERS, G., Lenhard, J. and Shinn, T. (2006). Computer simulation: Practice, epistemology, and social dynamics. In G. K. Johannes Lenhard and Terry Shinn (Eds.), *Simulation: Pragmatic construction of reality; sociology of the sciences yearbook* (pp. 3–22). Dordrecht, The Netherlands: Springer.

Conclusion

5.1 Based on a survey of 279 published articles this article portrayed the state-of-the-art in ABM and identified key research directions. It has been conjectured that ABM is an immature method and that standard practices promoting effective ABM modeling are neither clearly established nor accepted. This survey seems to support that conjecture. The lack of maturity and standard practices in the ABM field is reflected by the lack of models that were completely validated, the lack of references to the complete model and what is accepted as publishable results. A remedy is that techniques, philosophies and methods need to be adopted from other simulation paradigms, or developed specifically for ABM, and these techniques, philosophies and methods need to be taught to those using ABM such that they can build more effective models.

B. Heath, R. Hill and F. Ciarallo, A survey of Agent-Based Modeling Practices (January 1998 to July 2008), Journal of Artificial Societies and Social Simulation (2009).

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Different fields investigated by agent based models (1998-2008)

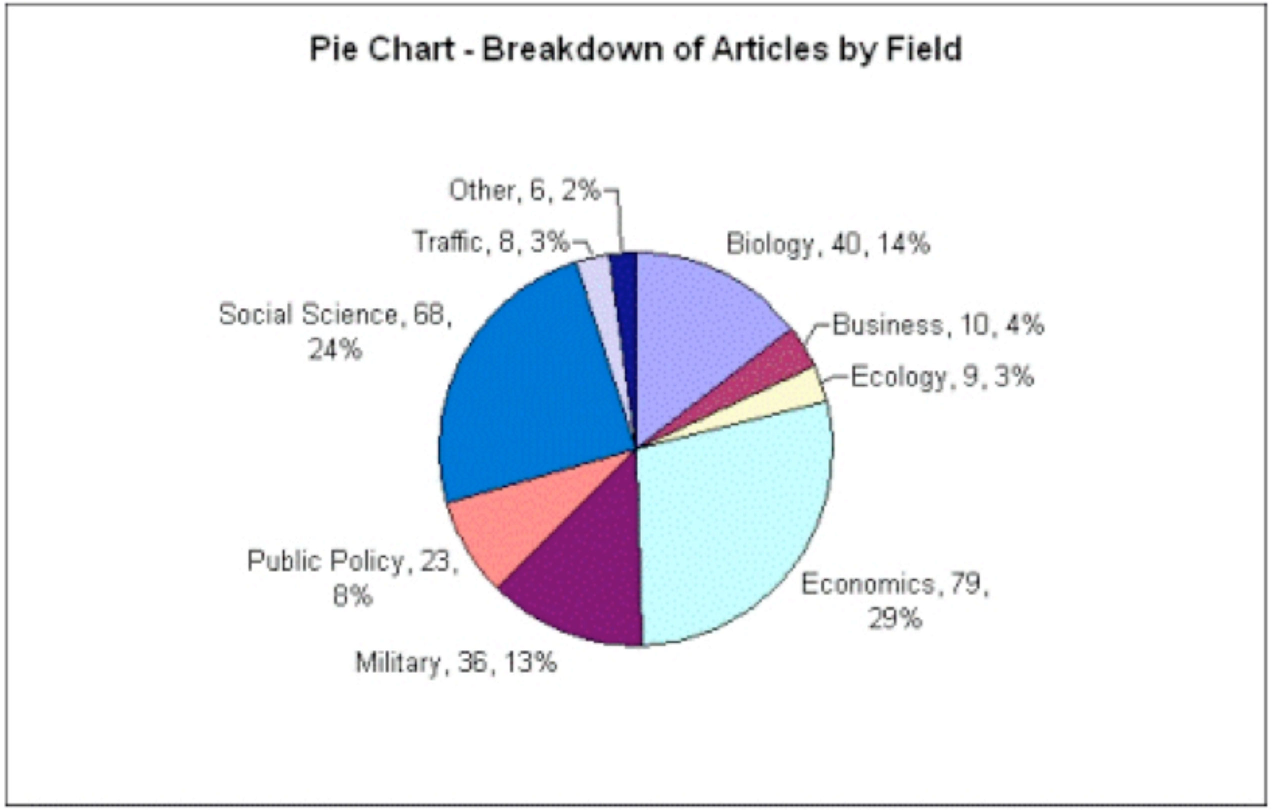

Figure 6. Breakdown of Articles by Field

A classification of the simulation purpose

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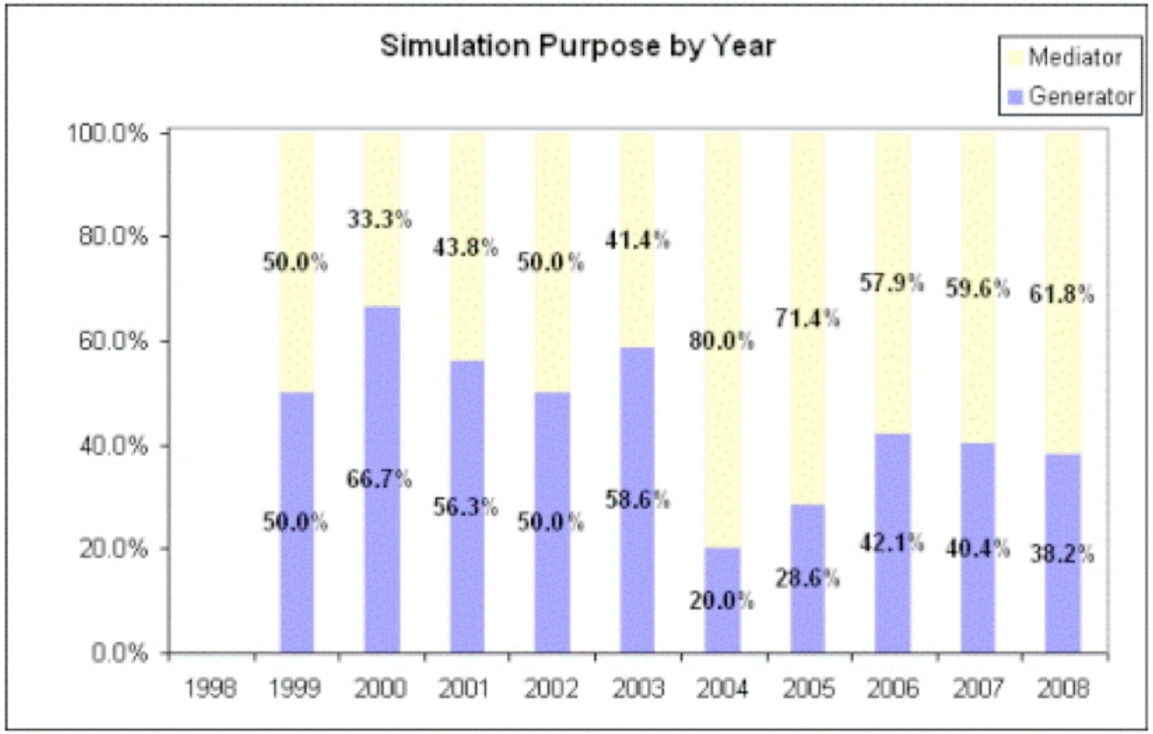

Figure 7. Simulation Purpose by Year

Agent based models in finance

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Kim – Markowitz model and the crash of 1987

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Two types of investors:

- Rebalancers
- Portfolio insurers

Two assets, stocks and cash (interest rate = 0)

The wealth of each agent at time t is

$w_t = q_t \, p_t + c_t$

q_t is the volume of stock

p_t is the price of the stock

c_t is the cash available

Rebalancers

Rebalancers are defined by the following target:

$$q_t \, p_t = c_t = 0.5 \, w_t$$

Rebalancing strategy has a stabilizing effect on the market: increasing prices induce rebalancers to raise their supply or reduce their demand

Portfolio insurers

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Portfolio insurers follow a strategy intended to guarantee a minimal level of wealth (the so-called “floor” f)

A classical strategy is the constant proportion portfolio insurance (CPPI) proposed by Black and Jones[˩]

The target of portfolio insurers is :

$$q_t\, p_t = k\, s_t = k\,(w_t - f)$$

where s_t is the “cushion” and k is chosen greater than 1.
The floor f is constant over the duration of the insurance plan.

[˩] Black F and Jones RC, Simplifying portfolio insurance J. Portfolio Manag. **14** 48–51 (1987)

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Portfolio insurers

In the case of price increase investors increase their demand.

In the case of price decrease stock position of the investor is reduced and therefore demand is reduced.
For falling prices the fraction of risky asset in the investor’s portfolio goes to zero.

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Kim - Markowitz model

Stock price and trading volume evolve endogenously according to supply and demand. Trading is discrete in time. Each investor reviews her/his portfolio at random interval. Each investor performs an individual forecast according to the current supply and demand situation.

$$
p^i_{\mathrm{est},t} = \begin{cases}
1.01 \max(p^1_{\mathrm{ask},t}, \cdots, p^n_{\mathrm{ask},t}), \\
\quad \text{if } p^i_{\mathrm{bid},t} = 0 \quad \text{for all} \quad i = 1, \ldots, n \\
\quad \text{and } p^i_{\mathrm{ask},t} \neq 0 \quad \text{for at least one} \quad i, \\
0.99 \min(p^1_{\mathrm{bid},t}, \cdots, p^n_{\mathrm{bid},t}) \quad \text{for} \quad p^i_{\mathrm{bid}} > 0, \\
\quad \text{if} \quad p^i_{\mathrm{ask},t} = 0 \text{ for all} \quad i = 1, \ldots, n \\
\quad \text{and} \quad p^i_{\mathrm{bid},t} \neq 0 \text{ for at least one} \quad i, \\
0.5[\max(p^1_{\mathrm{ask},t}, \cdots, p^n_{\mathrm{ask},t}) + \min(p^1_{\mathrm{bid},t}, \cdots, p^n_{\mathrm{bid},t})] \\
\quad \text{for} \quad p^i_{\mathrm{bid}} > 0, \\
\quad \text{if } p^i_{\mathrm{ask},t} \neq 0 \quad \text{for at least one} \quad i \\
\quad \text{and} \quad p^i_{\mathrm{bid},t} \neq 0 \quad \text{for at least one} \quad i, \\
p_{t-1}, \quad \text{if } p^i_{\mathrm{ask},t} = 0 \quad \text{and} \quad p^i_{\mathrm{bid},t} = 0 \quad \text{for all} \quad i = 1, \ldots, n,
\end{cases} \tag{6}
$$

If only asks exist the price is 101% of the highest ask.

If only bids exist the price is 99% of the lowest bid.

If both bids and asks exist the price is the average of the highest ask and the lowest bid.

If no bids or asks exist the price is the previous price.

In the case the estimated ratio between stocks and assets (for rebalancers) or between stocks and cushion (for portfolio insurers) is higher than the target ratio, the investor will place a sale order with

$$p^{i}_{bid,t} = 0.99\, p^{i}_{est,t}$$

Conversely, he/she will place a buy order

$$p^{i}_{ask,t} = 1.01\, p^{i}_{est,t}$$

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Kim – Markowitz simulations[1]

Price

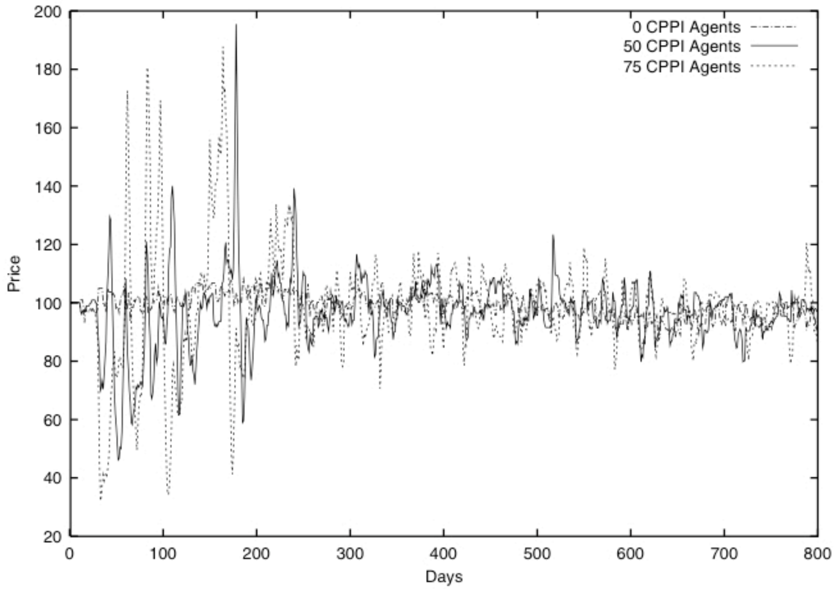

Figure 1. The daily development of prices (total number of agents: 150).

Volume

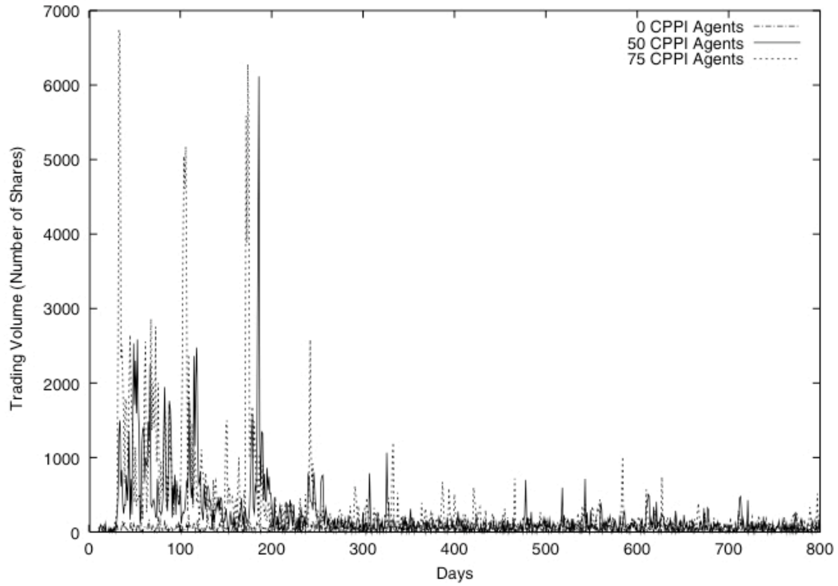

Figure 2. The daily development of trading volume (total number of agents: 150).

150 agents (number of CPPI agents variable)

[1]E.Samanidou, E. Zschischang, D.Stauffer, T.Lux, Agent-based models of financial markets, Rep. Prog. Phys. 70 409-450 (2007)

Kim – Markowitz simulations[*]

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Volatility

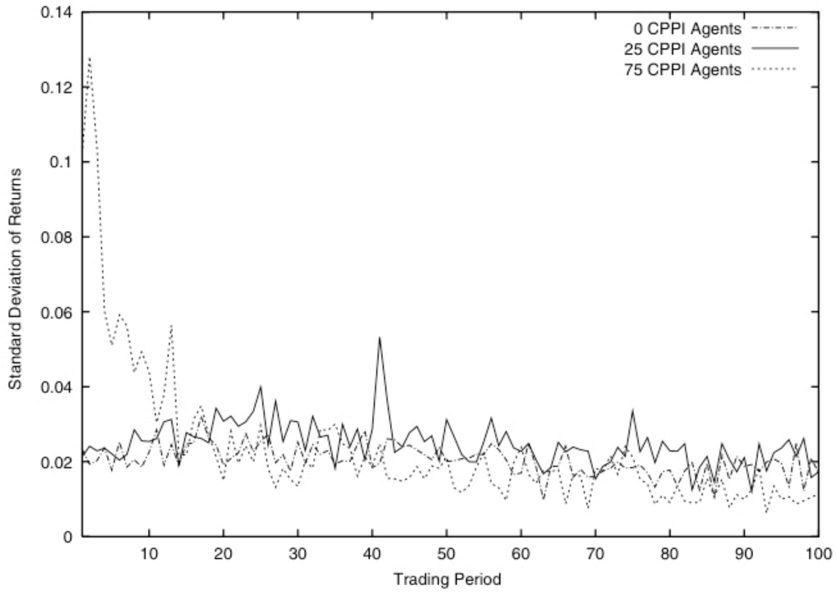
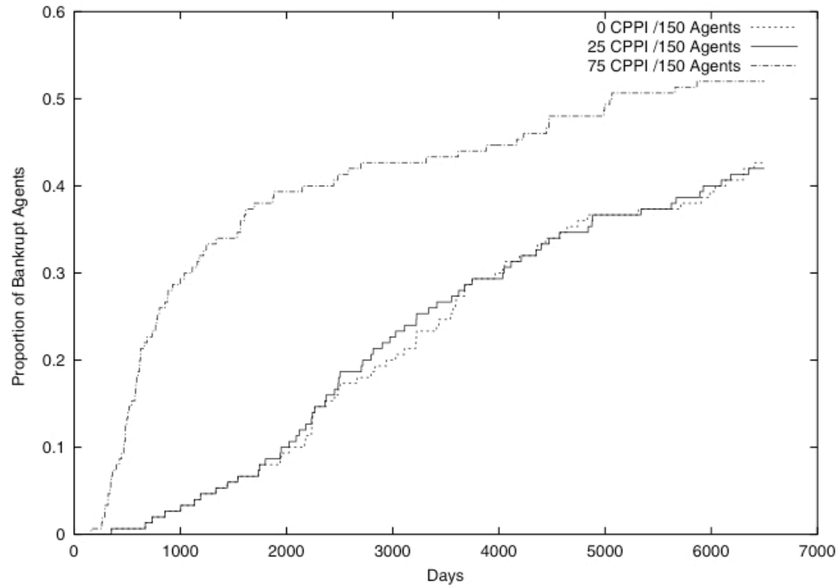


Figure 3. The standard deviation of daily returns per trading period (total number of agents: 150).

Percent of bankrupt investors



The basic result of this agent based model is the demonstration of the destabilizing potential of portfolio insurance strategies.

[*]E.Samanidou, E. Zschischang, D.Stauffer, T.Lux, Agent-based models of financial markets, Rep. Prog. Phys. 70 409-450 (2007)

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Levy – Levy – Solomon (1994)

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The model contains an ensemble of interacting investors which are using a utility maximization scheme.

At each period of time each investor *i* needs to divide up his entire wealth *W(i)* into stock shares and bonds.

X(i) is the fraction of wealth invested in stocks

$$W_{t+1} = \underbrace{X(i)W_t(i)}_{\text{wealth in stocks}} + \underbrace{\left(1 - X(i)\right)W_t(i)}_{\text{wealth in bonds}}$$

with $0.01 < X(i) < 0.99$

The number of investors n and the supply of shares N_A are fixed

M. Levy, H. Levy, and S. Solomon. A microscopic model of the stock market: Cycles, booms, and crashes. Economics Letters, 45:103–111, 1994.

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At the beginning investors possess the same wealth and the same fraction of stocks. They also possess the same utility function.

Bonds are assumed to be riskless.

Stock return is defined as $R_t = \frac{p_t - p_{t-1} + D_t}{p_{t-1}}$

where p_t is the price of the stock and D_t is the dividend.

The utility function is a logarithmic utility function

$$U(W) = \ln(W)$$

This implies constant relative risk aversion. The optimal proportion of wealth invested in stocks will therefore be independent of the wealth.

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Investors form their expectations of future returns on the basis of past observations.

The used records are the past k stock returns.

All investors with the same memory k form an investor group G.

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For the investor group with *k* memory

$$EU = \frac{1}{k}\left[\sum_{j=t}^{t-k+1} \ln\left[\left(1 - X_G(i)\right)W_t(i)(1+r) + X_G(i)W_t(i)\left(1+R_j\right)\right]\right]$$

$$f\left(X_G(i)\right) = \frac{\partial \mathrm{EU}\left(X_G(i)\right)}{\partial X_G(i)} = \sum_{j=t}^{t-k+1} \frac{1}{X_G(i) + \dfrac{1+r}{R_j - r}}$$

In this way $X_G(i)$ is obtained. To model idiosyncratic factors a normally distributed random variable ε_i is added to $X_G(i)$

From the aggregation of the total demand compared with the constant supply a new "equilibrium" price is determined.

This allows to compute the new return which is therefore used for the new estimation of the next price value.

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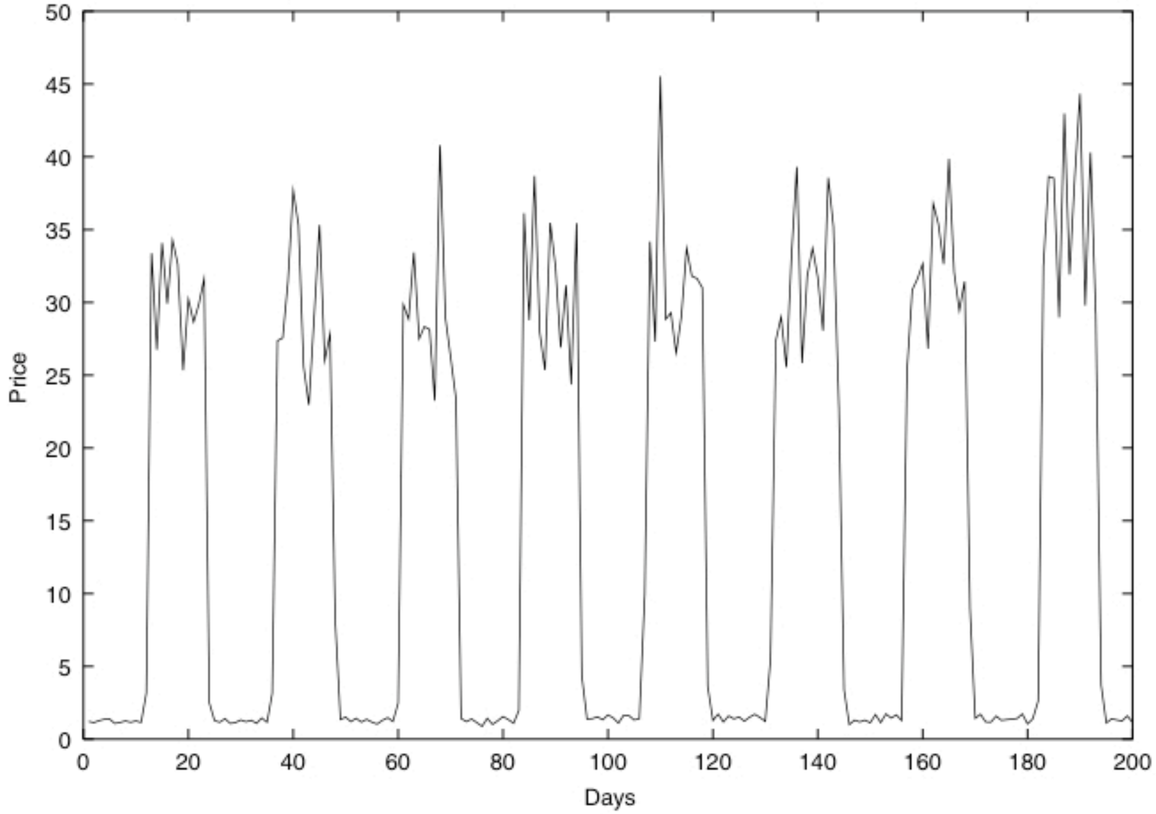

Figure 8. With only one type of traders and a logarithmic utility function, the typical outcome of the Levy–Levy–Solomon model is a cyclic development of stock prices with periodic booms and crashes. Our own simulations produced all the visible patterns emphasized in [117, 118, 121].

The model shows cyclic alternation of bubbles and crashes and also "chaotic phases".

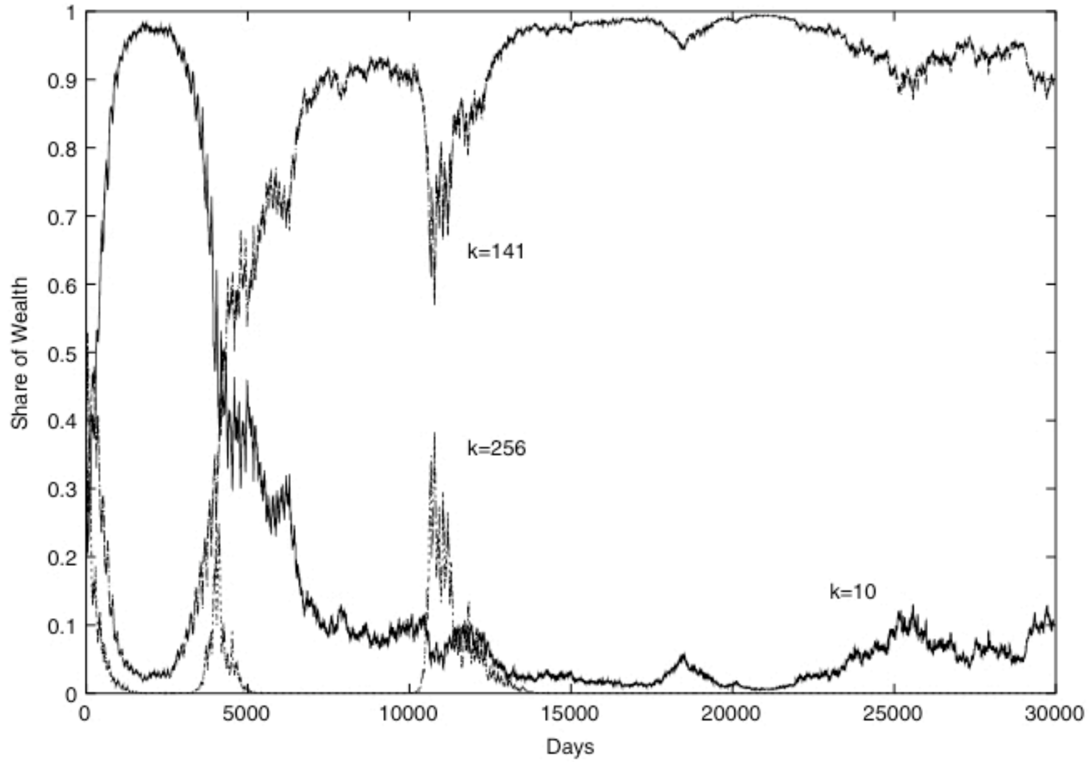

Figure 9. Development of the distribution of wealth with three groups characterized by $k = 10$, 141 and 256, respectively. Depending on the initial conditions, either the group with $k = 256$ or the group with $k = 141$, as in the present case, may happen to dominate the market.

The success of different groups of investors evolves in time and depends on the initial conditions. The time evolution is therefore non-ergodic.

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Stylized facts are investigated

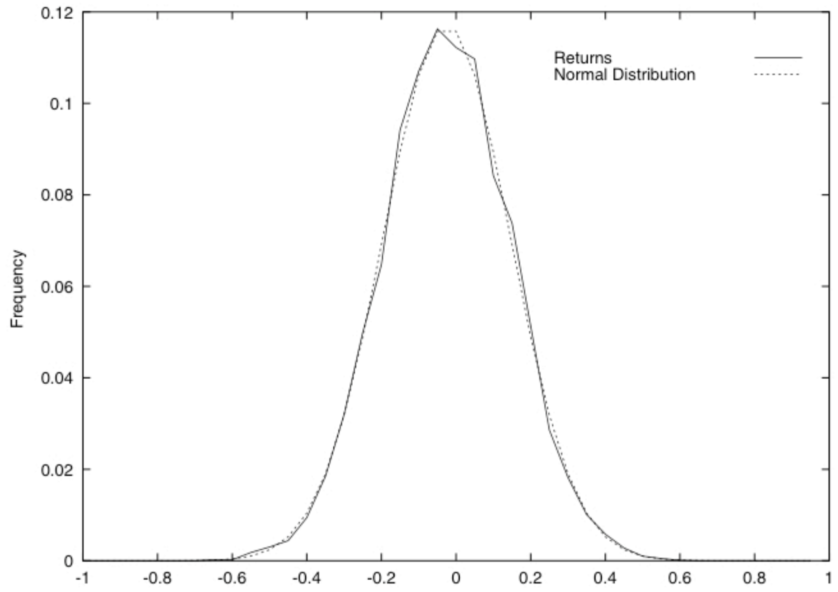

Figure 10. Distribution of returns from a simulation with six groups characterized by memory spans $k = 10, 36, 141, 193, 256$ and 420. This is an example with a stock price development described as 'chaotic' in [121]. However, it seems that the result is rather similar to pure randomness. The histogram is drawn for 20 000 observations after an initial transient of 100 000 time steps. The close similarity to the normal distribution is confirmed by statistical measures: Kurtosis is 0.043 and skewness is -0.003. This yields a Jarque–Bera statistic of 1.55 which does not allow the normal distribution to be rejected (significance is 0.46%). (The horizontal axis shows the dimensionless relative price changes.)

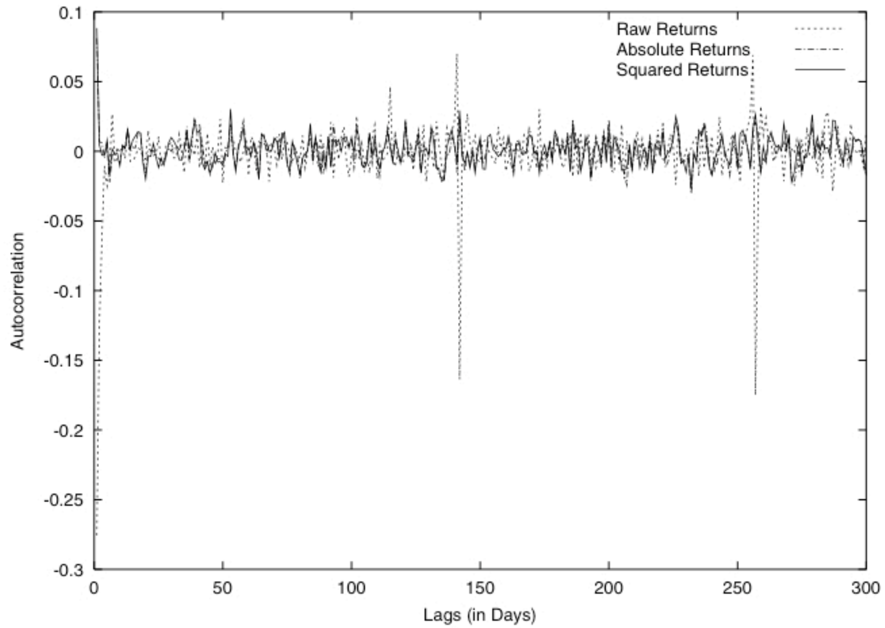

Figure 11. Autocorrelations of raw, absolute and squared returns from the simulation of a 'chaotic' case of the Levy–Levy–Solomon model. As can be seen, dependence in absolute and squared returns (as typical proxies for market volatility) is as weak as with raw returns themselves. The underlying scenario is the same as in figure 10.

In the "chaotic" regime the return pdf is Gaussian

Price return is uncorrelated and volatility cluster absent.

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The Santa Fe Artificial stock market (1997)

Arthur, W.B., Holland, J., LeBaron, B., Palmer, R., Tayler, P., 1997. Asset pricing under endogenous expectations in an artificial stock market. In: Arthur, W.B., Durlauf, S., Lane, D. (Eds.), The Economy as an Evolving Complex System II. Addison-Wesley, Reading, MA, pp. 15-44.

B. LeBaron, W. B. Arthur, and R. Palmer. The time series properties of an artificial stock market. Journal of Economic Dynamics and Control, 23:1487–1516, 1999.

There are two assets traded. A risk free bond paying a constant interest rate r_f (0.10) and a second risky stock paying a stochastic dividend following an autoregressive process

$$d_t = d_o + \rho\left(d_{t-1} - d_o\right) + \mu_t$$

with $d_o=10$, $\varrho=0.95$ and $\mu_t \sim N(0,\sigma^2_\mu)$

The agents (N=25) are assumed to be myopic of period 1 and characterized by constant absolute risk aversion (CARA).

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$\hat{E}_t$ is meaning the best forecast of agent *i* at time *t*

$$\hat{E}_t\left(-\exp\left(-\gamma W_{t+1}^{i}\right)\right)$$

with $$W_{t+1}^{i} = x_t^i\left(p_{t+1} + d_{t+1}\right) + \left(1 + r_f\right)\left(W_t^{i} - p_t x_t^i\right)$$

where x^i_t is the share demand which under Gaussian assumption for price and dividends is

$$x_t^i = \frac{\hat{E}_t^{i}\left(p_{t+1} + d_{t+1}\right) - \left(1 + r_f\right)p_t}{\gamma \hat{\sigma}^2_{p+d,i}}$$

$\hat{\sigma}^2_{p+d,i}$ is the forecast of conditional variance of p+d

Agents are homogeneous with respect to the risk aversion characterization.

It is therefore possible to obtain a homogeneous linear rational expectations equilibrium. Specifically, by assuming

$$p_t = f d_t + e$$

and imposing each agent to optimally hold one share at all times one obtains

$$f = \frac{\rho}{1 + r_f - \rho}$$

$$e = \frac{d_o (f + 1)(1 - \rho) - \sigma^2_{p+d}}{r_f}$$

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Heterogeneity in the agent based model

The homogeneous rational expectations equilibrium is

$$E\left(p_{t+1} + d_{t+1}\right) = \rho\left(p_t + d_t\right) + (1 - \rho)\left((1 + f)d_o + e\right)$$

Each agent is forecasting according to the linear equation

$$E^{\wedge}\left(p_{t+1} + d_{t+1}\right) = a\left(p_t + d_t\right) + b$$

with *a* and *b* parameters characterizing each agent

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By summarizing the model parameters are

1492 *B. LeBaron et al. / Journal of Economic Dynamics & Control 23 (1999) 1487–1516*

Table 1
Parameter values

Parameter	Simulation value
γ	0.5
$\bar{d}_o$	10
r_f	0.10
ρ	0.95
σ_μ^2	0.07429
σ_{p+d}^2	4.00
f	6.3333
e	16.6880
$(1-\rho)((1+f)\bar{d}_o + e)$	4.5011
a range	[0.7,1.2]
b range	[− 10.0,19.0]

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Achieving the best forecast

There is no role for imitative behavior but there is a learning process to select the best accessible forecasting.

Each agent has a 100 entry table to perform forecasts and uses it to estimate her/his best parameters a and b.

Agents build forecasts using “condition-forecast” rules a modification of Holland’s “condition-action” classifier system.

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For agent *i* the demand for the risky asset is

$$x_t^i(p_t) = \frac{E_t^{\wedge}(p_{t+1} + d_{t+1}) - (1 + r_f)p_t}{\gamma \sigma_{p+d}^{\wedge 2}}$$

By balancing the supply (which is fix) and demand

$$\sum_{i=1}^{N} x_t^i(p_t)$$

The next price value p_{t+1} is obtained in simulations

The model shows some of the stylized facts (absence of correlation, deviation from equilibrium price, correlation of volumes, ...) but it is crucially sensitive to model parameters.

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Lux – Marchesi (1999)

The economic background of the Lux-Marchesi model

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E. Zeeman, On the unstable behavior of stock exchange, J. of Math. Econ. 1, 39-49 (1974)

A. Beja and M. Goldman, On the dynamic behavior of prices in disequilibrium, J. of Finance 34, 235-247 (1980).

THE RATIONALITY OF THE FOREIGN EXCHANGE RATE[†]

Chartists, Fundamentalists, and Trading in the Foreign Exchange Market

By Jeffrey A. Frankel and Kenneth A. Froot*

American Economic Review 80, 181-185 (1990)

TABLE 2—TECHNIQUES USED BY FORECASTING SERVICES

Year	Total	Chartist	Fund.	Both
1978	23	3	19	0
1981	13	1	11	0
1983	11	8	1	1
1984	13	9	0	2
1985	24	15	5	3
1988	31	18	7	6

Source: *Euromoney*, August issues. *Notes:* Total = number of services surveyed; Chartist = number who reported using technical analysis; Fund. = number who reported using fundamentals models; and Both = number reporting a combination of the two. When a forecasting firm offers more than one service, each is counted separately.

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The model is capable of generating “bubbles” and volatility clustering.

The model has several variables and parameters.

N is the total number of agents;
n_c is the number of noise traders;
n_f is the number of fundamentalists;
n_+ is the number of optimistic noise traders;
n_- is the number of pessimistic noise traders;
p is the market price of the asset;
p_f is the fundamental price of the asset.

$$N = n_c + n_f$$

$$n_c = n_+ + n_-$$

T. Lux and M. Marchesi. Scaling and criticality in a stochastic multi-agent model of a financial market. Nature, 397:498–500, 1999.

Dynamical exchange of investors' strategies and mood

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Noise traders switch from pessimistic to optimistic and vice versa

The probability of these switches are $\pi_{+-}\ \Delta t$ and $\pi_{-+}\ \Delta t$

π_{ab} is the probability to switch from state *a* to *b*

$$\pi_{+-} = v_1 \frac{n_c}{N} \exp(U_1) \qquad U_1 = \alpha_1 x + \frac{\alpha_2}{v_1} \frac{dp}{dt} \frac{1}{p}$$

$$\pi_{-+} = v_1 \frac{n_c}{N} \exp(-U_1) \qquad x = (n_+ - n_-)/n_c$$

v1, α1, α2 are parameters controlling the frequency of changing opinion.

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Switches between the group of noise traders and the group of fundamentalists.

$$\pi_{+f} = v_2 \frac{n_+}{N} \exp\left(U_{2,1}\right) \qquad \pi_{f+} = v_2 \frac{n_f}{N} \exp\left(-U_{2,1}\right)$$

$$\pi_{-f} = v_2 \frac{n_-}{N} \exp\left(U_{2,2}\right) \qquad \pi_{f-} = v_2 \frac{n_f}{N} \exp\left(-U_{2,2}\right)$$

$U_{2,1}$ and $U_{2,2}$ depend on the difference between profit earned by investors using chartist and fundamentalist strategy.

$$U_{2,1} = \alpha_3 \left\{ \frac{r + \frac{1}{v_2}\frac{dp}{dt}}{p} - R - s \cdot \left| \frac{p_t - p}{p} \right| \right\} \qquad U_{2,2} = \alpha_3 \left\{ R - \frac{r + \frac{1}{v_2}\frac{dp}{dt}}{p} - s \cdot \left| \frac{p_t - p}{p} \right| \right\}$$

s<1, r is a nominal dividend and R is the average real return

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Price changes are modeled as endogenous responses to the excess demand.

The excess demand of chartists is

$ED_c = (n_+ - n_-)\, t_c$

where t_c is the average trading volume per transaction.

The excess demand of fundamentalists is

$ED_f = n_f\, \gamma\, (p_f - p)/p$

where γ is a trading parameter.

The price formation is therefore governed by

$$\frac{1}{p}\frac{dp}{dt} = \beta\left(\mathrm{ED}_c + \mathrm{ED}_f\right)$$

Log-changes of p_f are assumed to be Gaussian variables.

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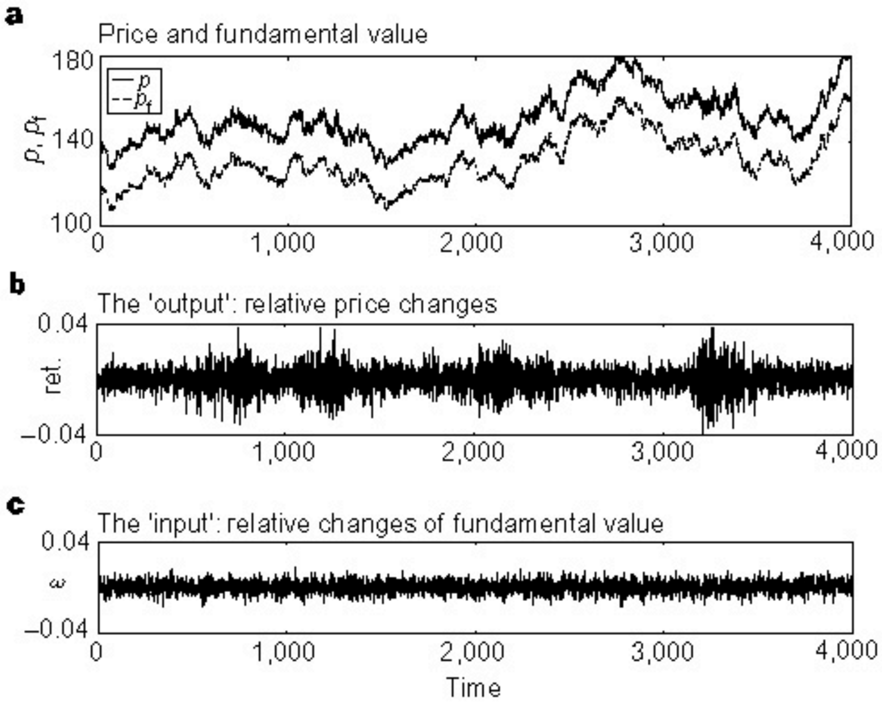
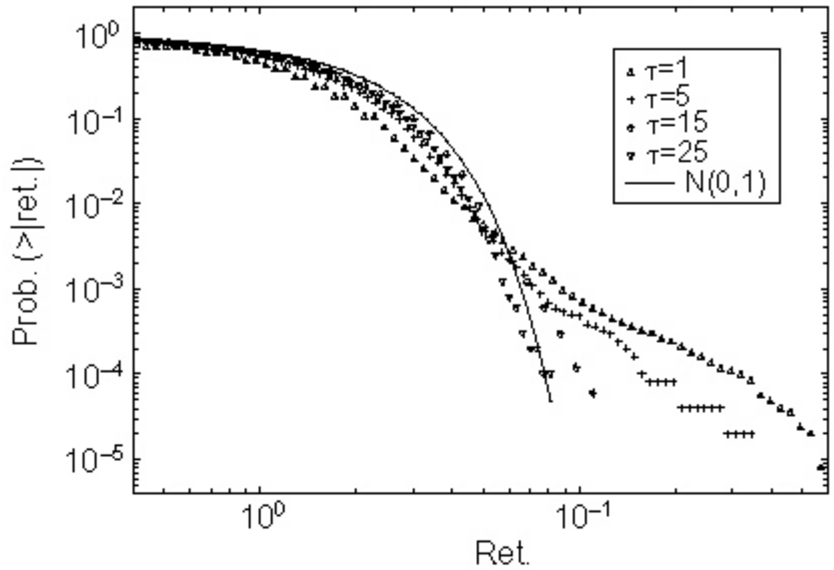
The model shows leptokurtosis of returns, volatility clustering and power law behavior of volatility scaling.





-Lux T, Marchesi M, Scaling and criticality in a
-stochastic multi-agent model of a financial
-market, Nature 397, 498-500 (1999)

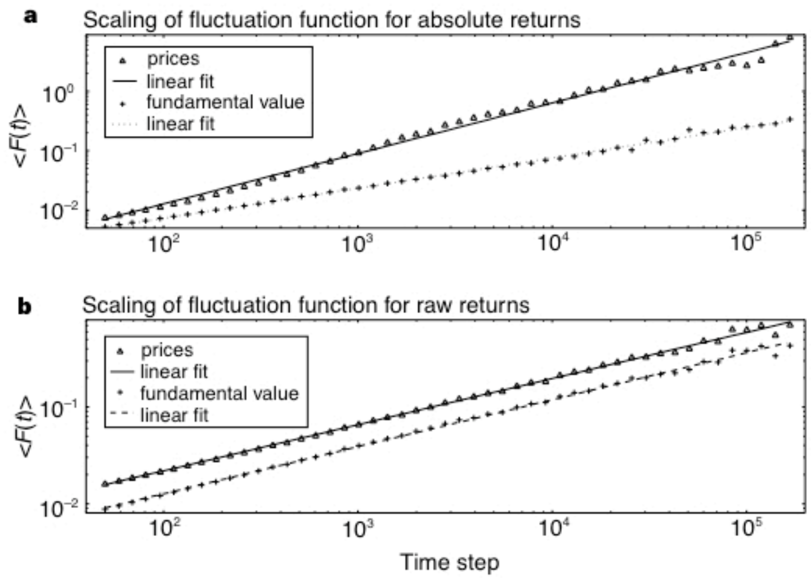


Figure 3 Estimation of self-similarity parameter H. **a**, Absolute returns; **b**, raw returns. Using the approach of Peng *et al.*[19] the exponent H was estimated from

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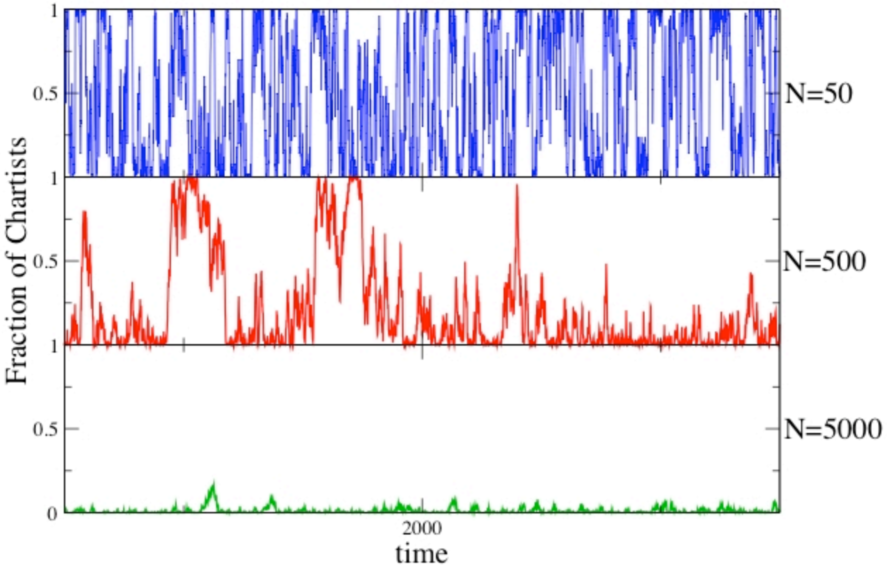
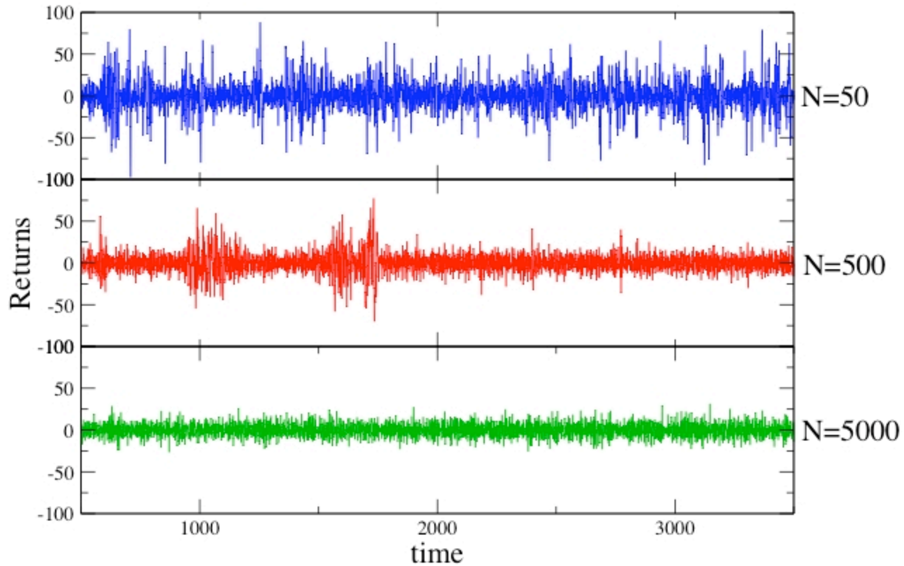
Excess demand linearly depends on the number of agents. This is not realistic but in spite of that the statistical profile of price return becomes Gaussian and the volatility clustering disappears.





V. Alfi, L. Pietronero, and A. Zaccaria, Minimal Agent Based Model for the Origin and Self-Organization of Stylized Facts in Financial Markets, arXiv:0807.1888v1 (2008).

Order book stylized facts

Average volume within the order book at a given time

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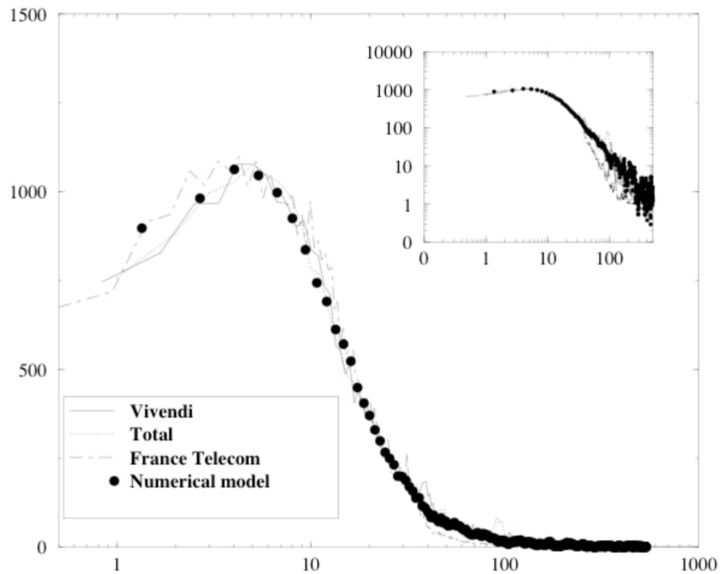
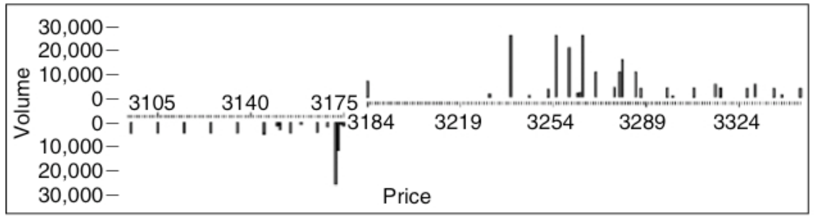


Figure 2: Average volume of the queue in the order book for the three stocks, as a function of the distance Δ from the current bid (or ask) in a log-linear scale. Both axis have been rescaled in order to collapse the curves corresponding to the three stocks. The thick dots correspond to the numerical model explained below, with $\Gamma = 10^{-3}$ and $p_m = 0.25$. Inset: same data in log-log coordinates.



Bouchaud, J. -P., M. Mezard, and M. Potters, Statistical properties of the stock order books: empirical results and models, Quantitative Finance, 2002, 2(4), 251–256.

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The cumulative distribution shows a quite robust power-law behavior

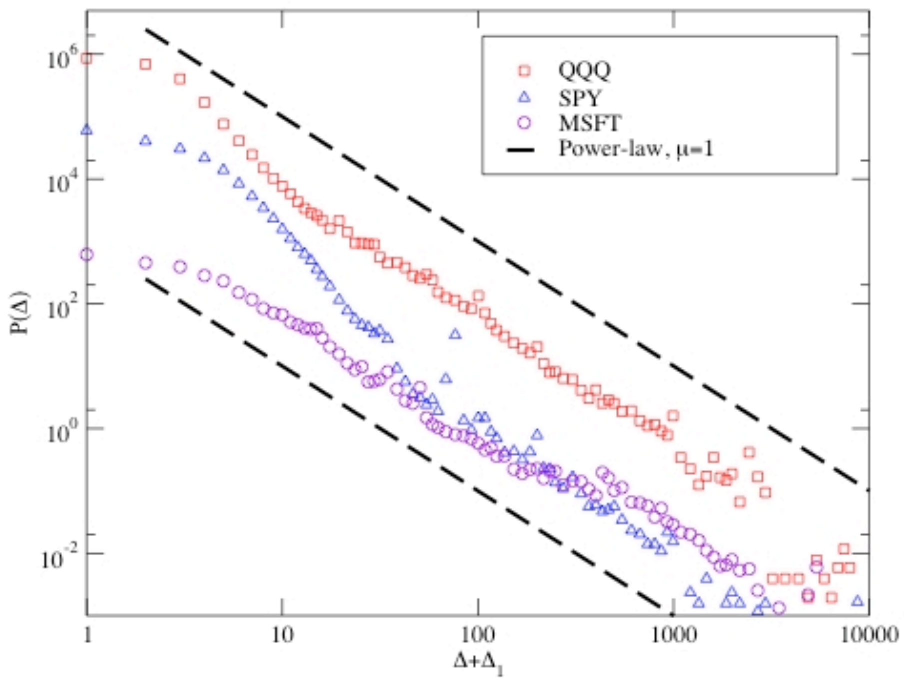

Figure 1: Cumulative distribution of the position Δ of incoming orders, as a function of $1 + \Delta$ (in ticks), for QQQ, SPY and MSFT. The dashed lines correspond to $\mu = 1$.

M. Potters, J.P. Bouchaud, More statistical properties of order books and price impact, Physica A, 324, 133 (2003).

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By assuming a completely random (Poissonian) flux of orders one can study the price formation mechanism of a double auction market. This approach has been described as zero-intelligence models.

Allocative Efficiency of Markets with Zero-Intelligence Traders: Market as a Partial Substitute for Individual Rationality

---

Dhananjay K. Gode and Shyam Sunder

Carnegie Mellon University

We report market experiments in which human traders are replaced by "zero-intelligence" programs that submit random bids and offers. Imposing a budget constraint (i.e., not permitting traders to sell below their costs or buy above their values) is sufficient to raise the allocative efficiency of these auctions close to 100 percent. Allocative efficiency of a double auction derives largely from its structure, independent of traders' motivation, intelligence, or learning. Adam Smith's invisible hand may be more powerful than some may have thought; it can generate aggregate rationality not only from individual rationality but also from individual irrationality.

The Journal of Political Economy 101, 119-137 (1993)

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By calling S the bid-ask spread and by assuming a Poissonian flow of limit (ρ arrival rate per unit share per unit time), market orders (μ arrival rate per unit time) and cancellations (δ arrival rate per unit time), Farmer and
co-workers obtained a relation between the expected spread and the order flux parameters

$$E[S] = \frac{\mu}{\rho} F\left(\frac{\sigma\delta}{\mu}\right)$$

where σ is the number of share per order present at the order book and *F(u)* is a monotonically increasing function empirically approximated as $F(u) \approx 0.28 + 1.86 u^{3/4}$

Daniels et al, Quantitative Model of Price Diffusion and Market Friction Based on Trading as a Mechanistic Random Process, PRL 90, 108102 (2003).

Farmer, J. D., P. Patelli, and I. Zovko, The predictive power of zero intelligence in financial markets, PNAS, 2005, 102(6), 2254–2259.

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As a first order approximation empirical results are described by the previous relation but several important aspects are not modeled in terms of zero-intelligence approaches.

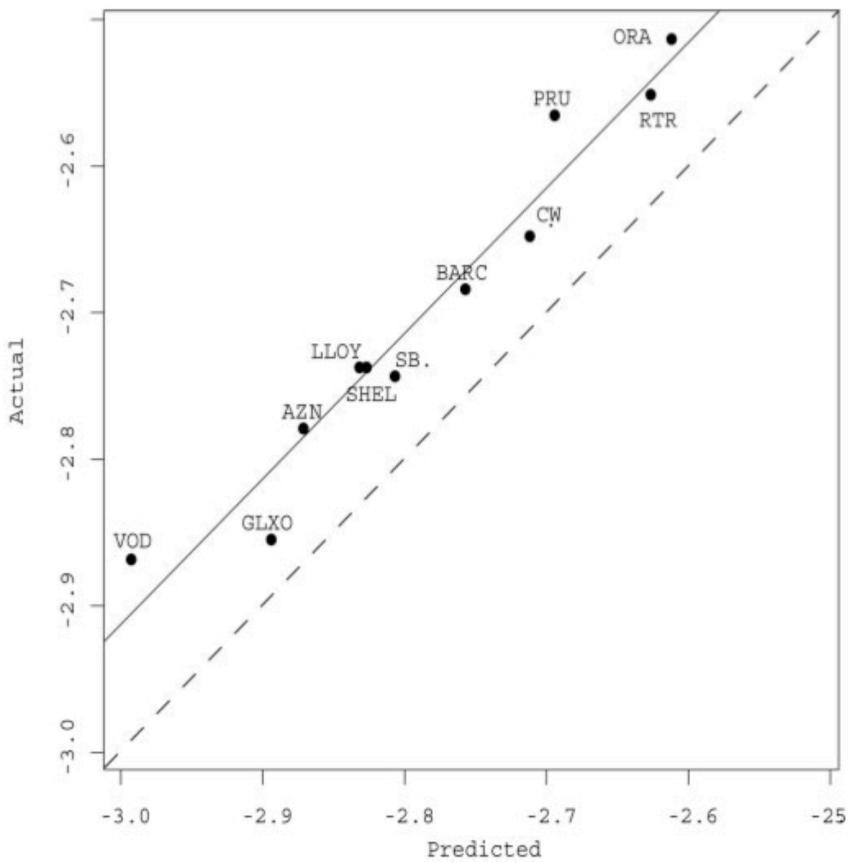


Fig. 2. Regressions of predicted values based on order flow using Eq. **1** vs. actual values for the log spread. The dots show the average predicted and actual value for each stock averaged over the full 21-mo time period. The solid line is a regression; the dashed line is the diagonal representing the model's prediction, with $A = 1$ and $B = 0$.

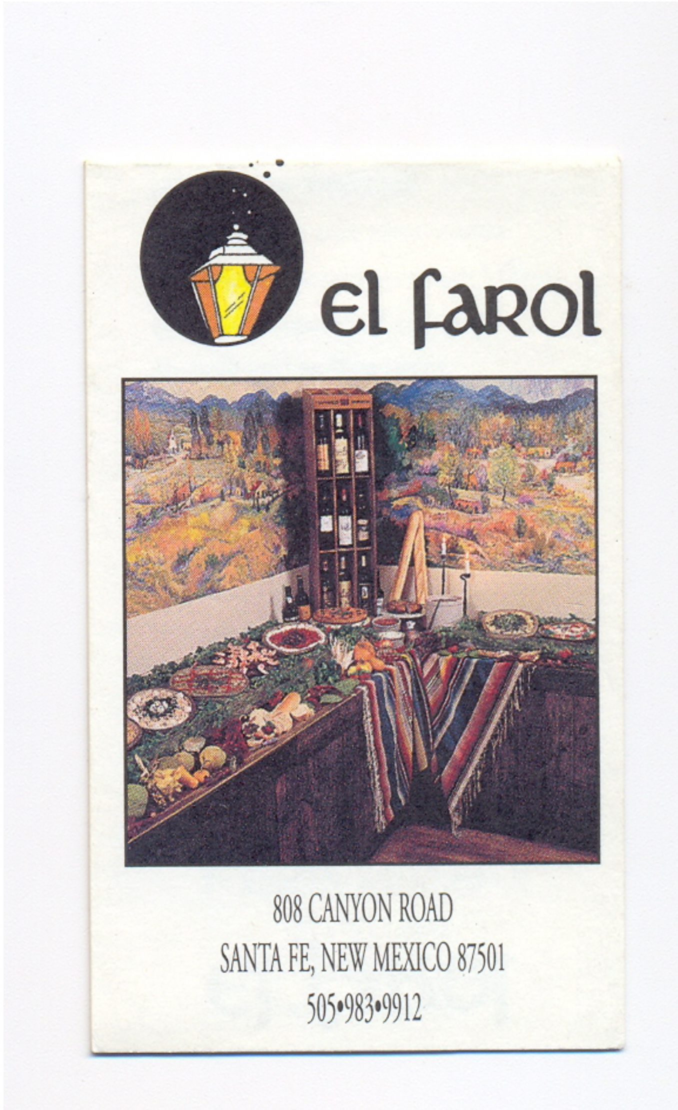
$$E[S] = \frac{\mu}{\rho} F\left(\frac{\sigma\delta}{\mu}\right)$$

Statistical physics models: The El Farol bar model and the Minority Game

OCS

El Farol Bar problem†

This problem was originally posed as an example of inductive reasoning in a scenario of bounded rationality.



†W.B. Arthur, Am. Econ. Assoc. Papers and Proc. 84, 406 (1994)

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At a given time, N people decide independently whether to go to a bar (El Farol)

Space is limited and the bar is enjoyable if it is not too crowded

The agent feel itself satisfied if the attendance at the bar is aN with $a<1$ otherwise he/she is unhappy

El Farol

At El Farol our cuisine is Spanish. We feature a full selection of entrées and vegetarian cuisine. Tapas (Spanish appetizers) are our speciality. *Paella*, saffron rice with seafood, chicken and chorizo is spectacular.

Cold Tapas Include

Ceviche
Shrimp and sun-dried tomato with herbed goat cheese
Curried chicken
Roasted, marinated sweet bell peppers
Caesar Salad

Hot Tapas Include

Grilled shrimp with romesco sauce
Fried calamare
Shrimp Ajillo, sauteed and spicy
Sauteed wild mushrooms
Grilled duck with fig and orange sauce
Grilled salmon with Aliolio (garlic sauce)
Pasta with pinon and manchego cheese

El Farol, the place where Santa Feans and visitors come for fine dining and fun.

Full Service Bar
Lunch and Dinner Daily
Live Entertainment Nightly
Most Major Credit Cards Accepted

Reservations Recommended
505 • 983 • 9912
Toll-free: 888-El Farol
Fax: 505-988-3823

A rational deductive approach induces frustration

In fact, if all agents were completely rational and sharing absolutely the same information they would choose all the same action.

- If they all go the bar will be crowded and they will be unhappy

- If they all stay home the bar will be empty and they will miss an opportunity

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By performing numerical simulations W.B. Arthur was able to show that by assuming inductive rather than deductive reasoning of heterogeneous agents the system reached a dynamical equilibrium where the average attendance is *aN* with a fluctuation of the order of *(1-a)N*

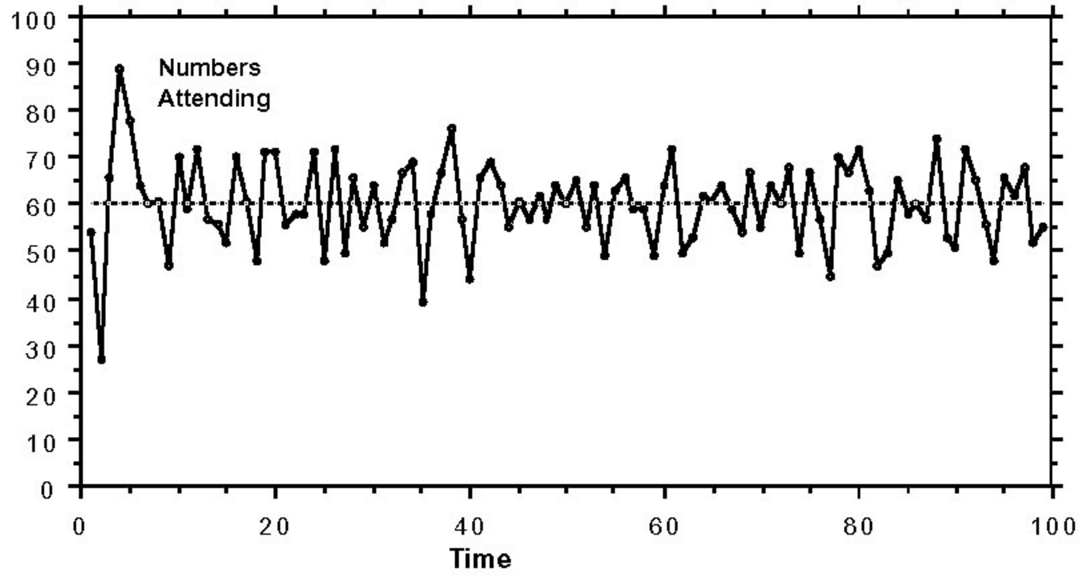

FIGURE 1. BAR ATTENDANCE IN THE FIRST 100 WEEKS.

The ingredients of the **El Farol** model are

- Many (N>>1) interacting agents;

- Interaction is through the aggregate bar attendance, i.e. of the mean field type.

- The system of n agents is frustrated, in the sense that there is not a unique winning strategy in the problem

- Quenched disorder is present, since agents use different predictors to generate expectation about the future

Agent based models investigated with statistical mechanics tools

A. De Martino and M. Marsili, Statistical mechanics of socio-economic systems with heterogeneous agents, J. Phys. A 39, R465-R540 (2006)

The minority game

In order to formalize the El Farol model, Challet and Zhang gave a precise mathematical definition of the El Farol bar model which they called minority game

In their model:

N agents take an action $a_i(t)$ deciding either to go ($a_i(t) = 1$) or to stay at home ($a_i(t) = -1$)

The agent who take the minority action win, whereas the majority looses

-Challet D, Zhang YC, Emergence of cooperation and organization in an evolutionary game, Physica A 246, 407-418 (1997)

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The minority game

After the decision, the total action *A(t)* is computed

$$A(t) = \sum_{i=1}^{N} a_i(t)$$

Agents choose their action by inductive reasoning.
Agents have limited analyzing power and they retain information only about the last *m* steps

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Number of possible strategies

Since there are 2^m possible inputs for each strategy, the total number of possible strategies for a given m is

$$2^{2^m}$$

Quenched disorder

Since the beginning of the game each agent has a set s of strategies randomly selected for the complete set of strategies.

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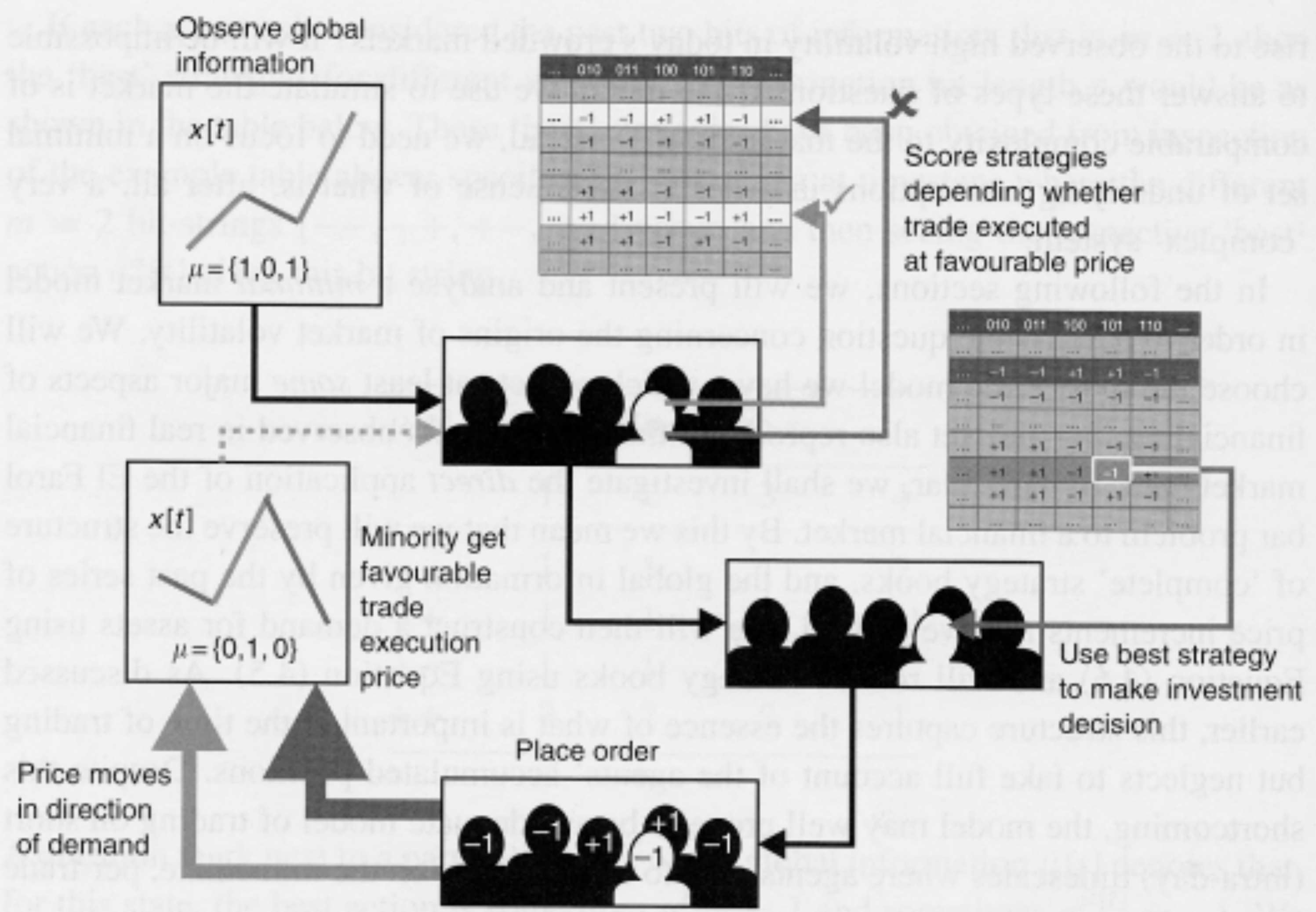

Fig. 4.8 Schematic diagram of the El Farol Market Model, which is a binary multi-agent game. This model represents our minimal market model.

- From N. Johnson's et al book on Financial market complexity, OUP

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Time evolution of the attendance

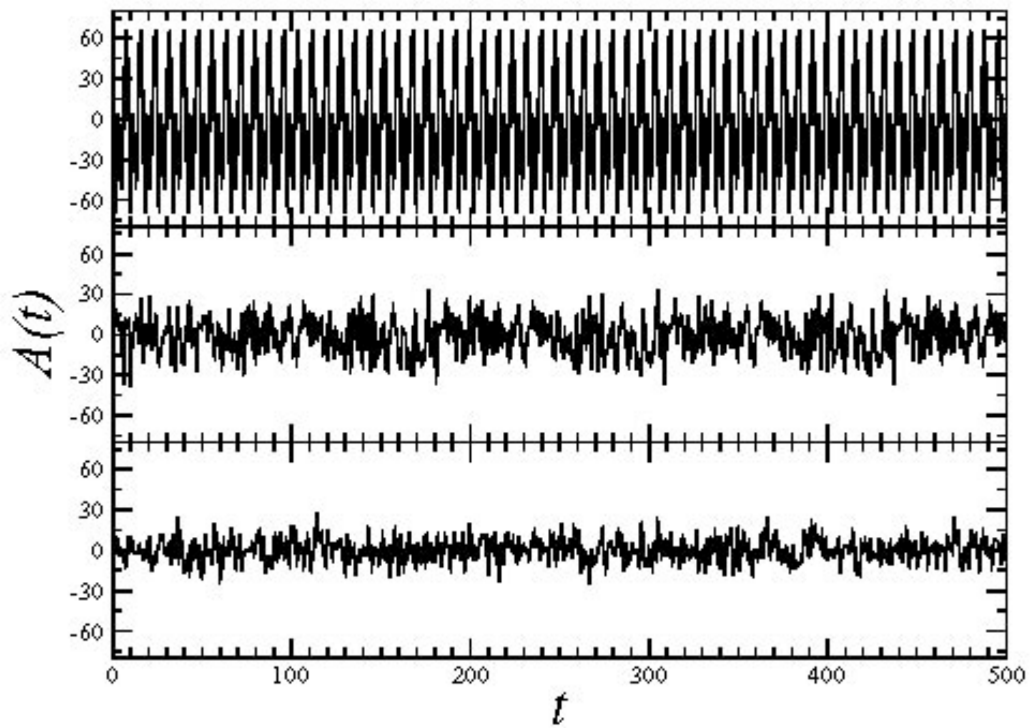

Figure 2: Time evolution of the attendance for the original MG with $g(x) = x$ and $N = 301$ and $s = 2$. Panels correspond to $m = 2, 7$ and $m = 15$ from top to bottom. Periodic patterns can be observed for $m = 2$ and $m = 7$.

A control parameter exists †

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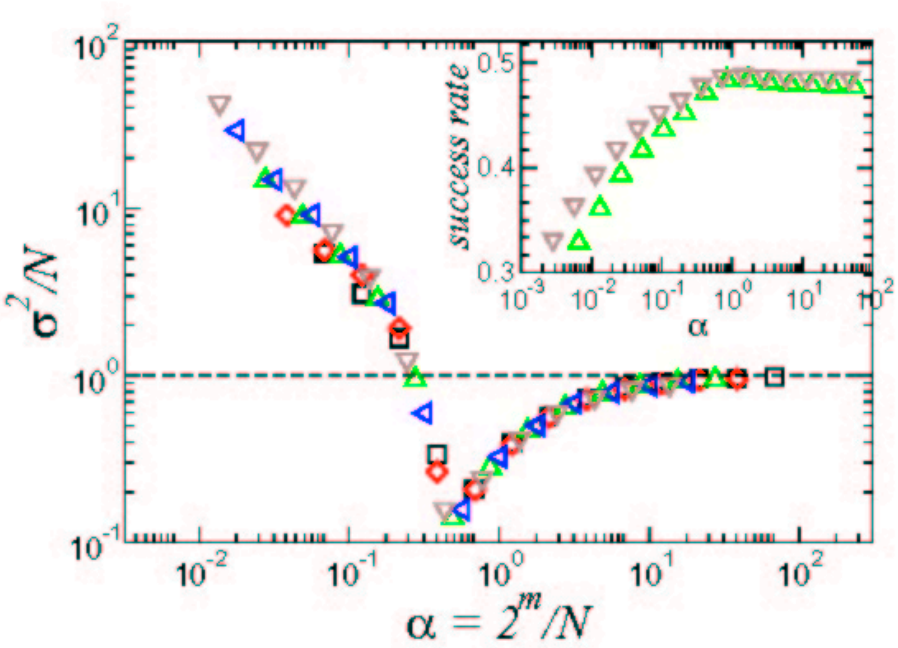

Figure 3: Volatility as a function of the control parameter $\alpha = 2^m/N$ for $s = 2$ and different number of agents $N = 101, 201, 301, 501, 701$ (□, ◊, △, ◁, ▽, respectively). Inset: Agent's mean success rate as function of α.

$$\alpha \frac{2^m}{N}$$

†R. Savit, R. Manuca and R. Riolo, PRL 82, 2203 (1999)

C

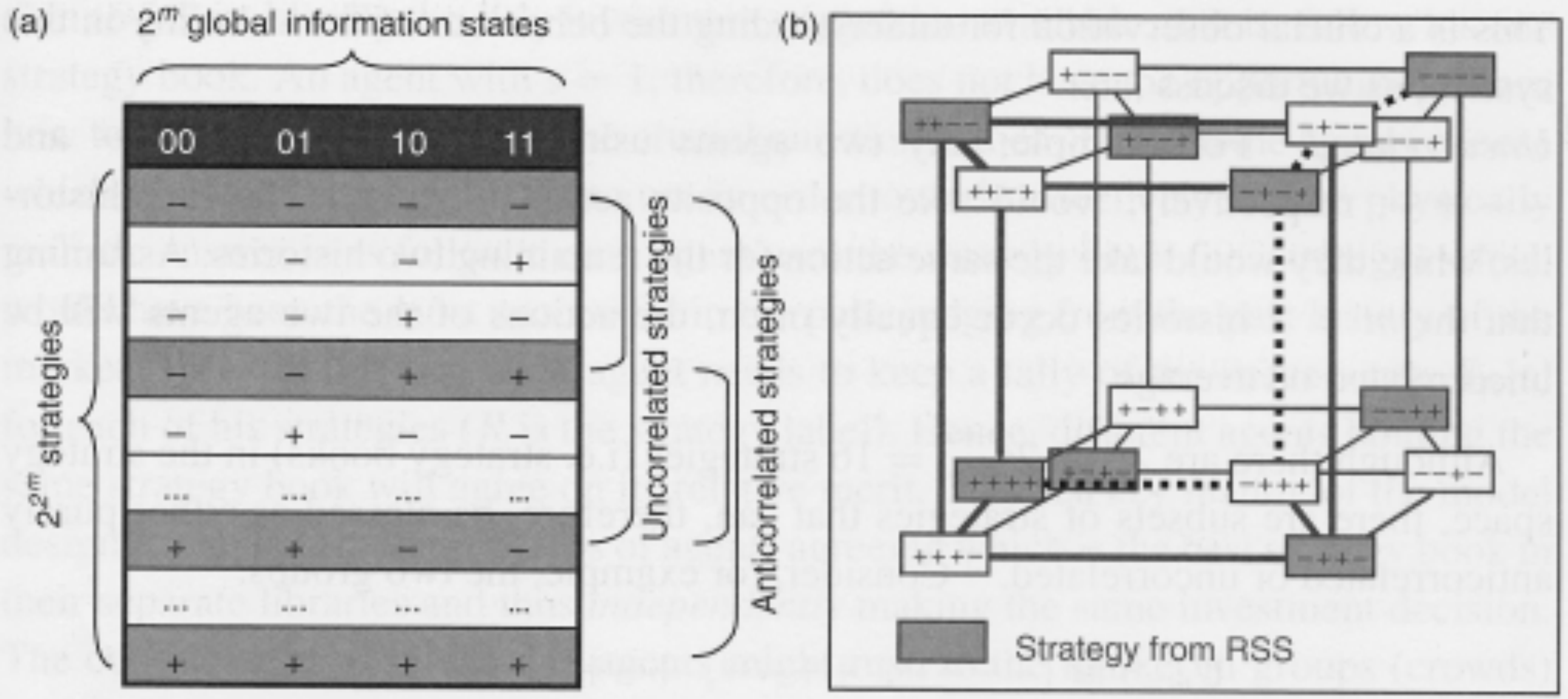

Fig. 4.3 'Strategy space' for $m = 2$. (a) Schematic representation of the $2^{2^m} = 16$ different strategies (i.e. strategy books). The greyed strategies belong to the Reduced Strategy Space (RSS) and are either totally uncorrelated or anticorrelated with respect to each other. There are $2^{m+1} = 8$ strategies in the RSS. (b) Representation of a $2^m = 4$ dimensional hypercube, which demonstrates the Hamming distance between strategies. The minimum number of edges linking strategies is the Hamming distance; for example, the dotted line shows a Hamming distance of 4 between strategies $----$ and $++++$.

Exploiting information in the game

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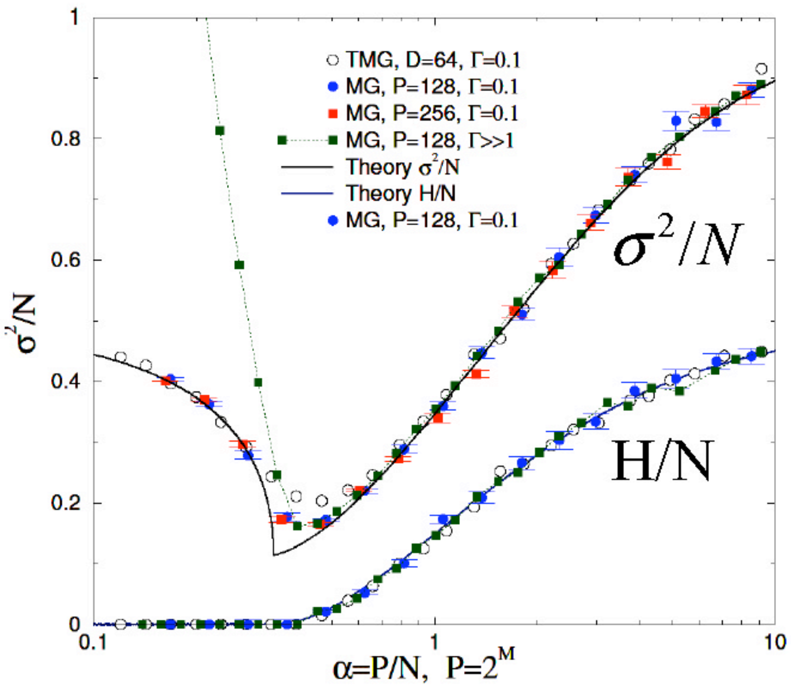


- **Predictability**

$$\langle A|\mu\rangle \neq 0 \quad \Rightarrow \quad \text{predictable}$$

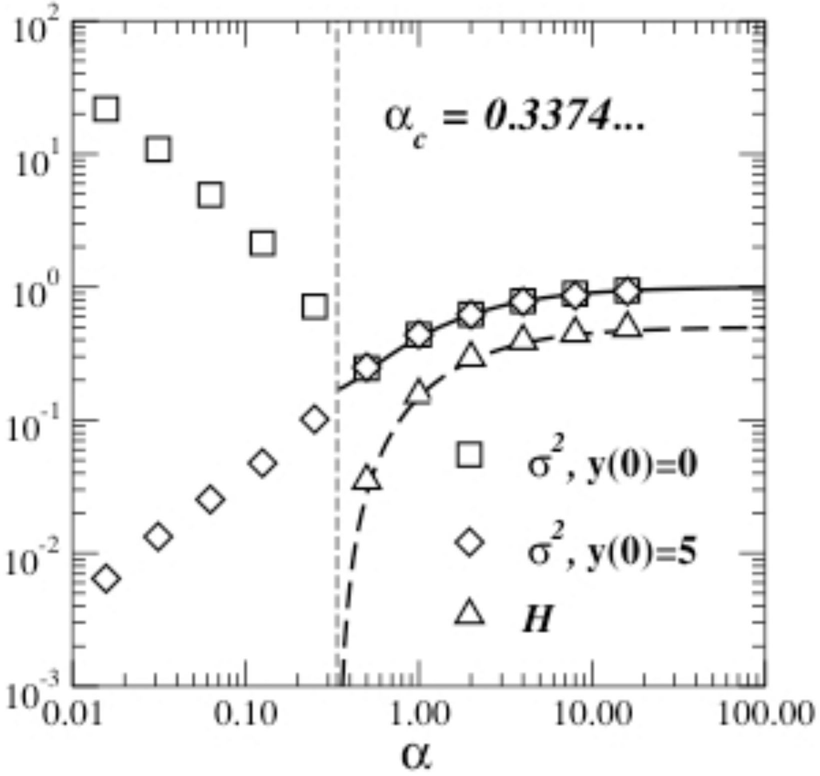
$$H = \frac{1}{P}\sum_{\mu=1}^{P}\langle A|\mu\rangle^2$$

- **Global efficiency (volatility)**

$$\sigma^2 = \langle A^2\rangle = -\sum_{i=1}^{N}\langle u_i\rangle$$

μ is the history of the past attendance

- This slide is from M. Marsili's presentation



There is a non-ergodic phase $\alpha < \alpha_c$ and an ergodic phase $\alpha > \alpha_c$

The variable H acts as an order parameter $$H = \frac{1}{2^m} \sum_{\mu=1}^{2^m} \langle A \mid \mu \rangle^2$$

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Including the impact of agent's own action

The score function Δ_i of strategy i is updated according to

$$\Delta_i(t+1) - \Delta_i(t) = -\Gamma A(t)/N$$

with $\Gamma > 0$ a constant.

By taking into account agent's own impact the score function is estimated as

$$\Delta_i(t+1) - \Delta_i(t) = -\frac{\Gamma}{N}\left[A(t) - \eta a_i(t)\right]$$

The term proportional to η describes agent's contribution to A(t).

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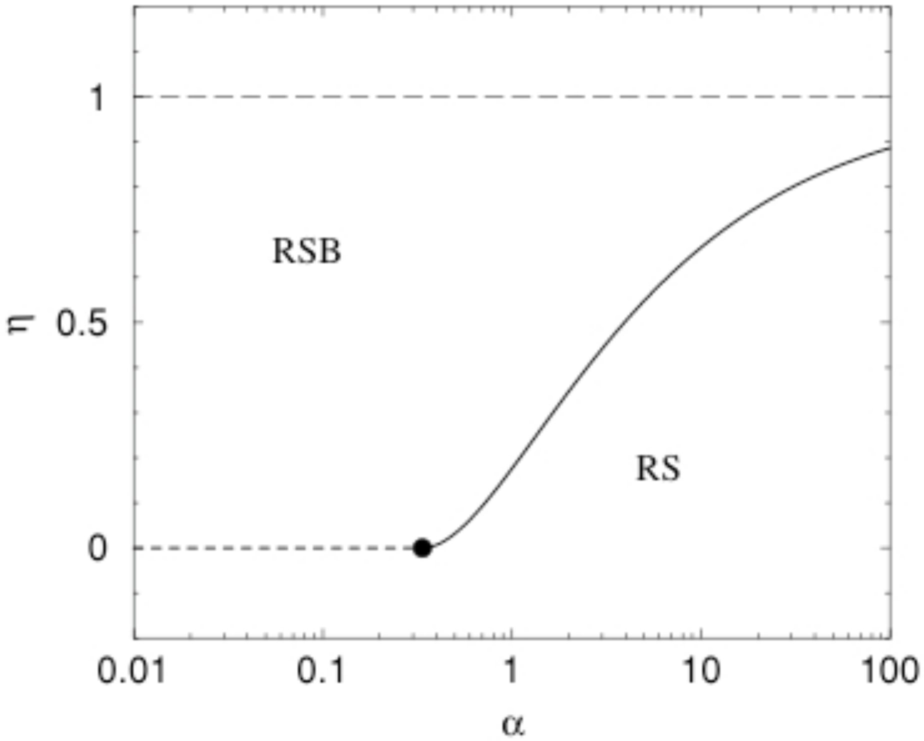

Phase diagram of the Minority Game in the (α,η) plane.
When η > 0 the transition is second order. When η=0
the transition is discontinuous.

It is worth noting the absence of a phase transition when η=1.

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Grand-canonical Minority Game and stylized facts

In this version of the Minority Game each agent has one quenched strategy and the possibility to choose whether to join the market or not.

By setting a different incentive to enter the market the agents are grouped into producers, who always enter the market, and speculators, whose trading frequency is a function of the expected fitness of the strategy.

$n_p = N_p/N$ and $n_s = N_s/N$ stand as the relative number of producers and speculators respectively.

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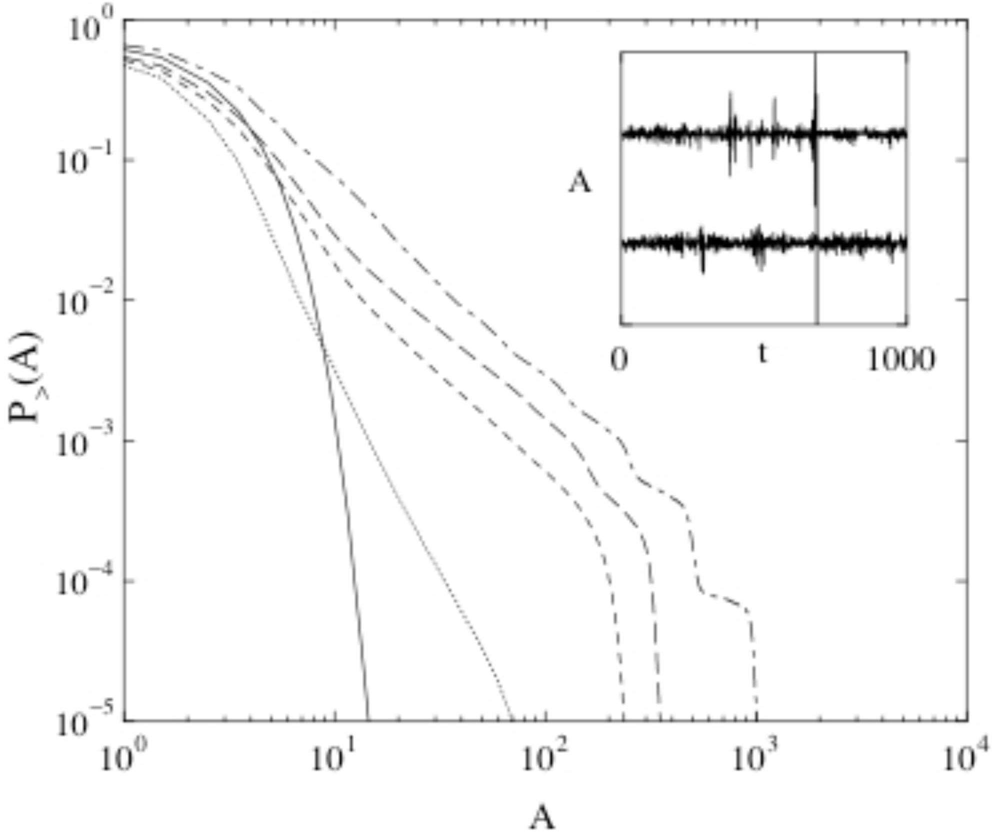
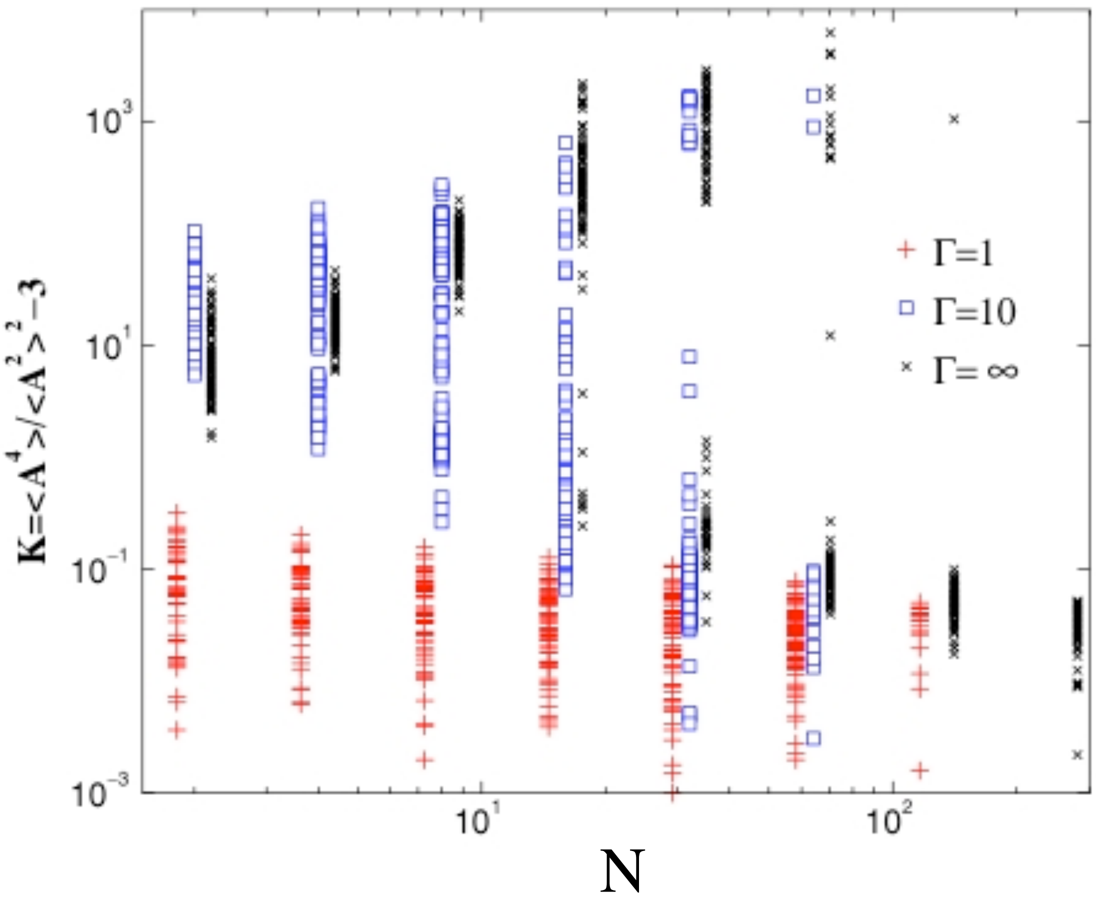
In the grand-canonical minority Game one observes
a dynamics of A(t) fluctuations similar to stylized facts





Kurtosis of A(t) in simulations with ε=0.01, n_s=70, n_p=1 and several values of N and Γ.

The kurtosis is decreasing when N is increasing!!!

Some reviews for reference:

B. Heath, R. Hill and F. Ciarallo, A survey of Agent-Based Modeling Practices (January 1998 to July 2008), Journal of Artificial Societies and Social Simulation (2009).

E.Samanidou, E. Zschischang, D.Stauffer, T.Lux, Agent-based models of financial markets, Rep. Prog. Phys. 70 409-450 (2007)

A. De Martino and M. Marsili, Statistical mechanics of socio-economic systems with heterogeneous agents, J. Phys. A 39, R465-R540 (2006)

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Thank you!

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