Surviving the Credit Crunch
new features for post-crisis pricing models

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Talk Outline

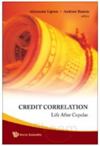
1. Multi-Name Credit Modelling: the case of CDOs
2. Counterparty Risk: pricing within Basel III framework
3. Interest-Rate Modelling: splitting of yield curves
4. Conclusions and Further Developments

Disclaimer

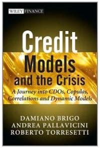
The opinions expressed in this work are solely those of the authors and do not represent in any way those of their current and past employers.

Reference Books – I

- Lipton A., and Rennie A. (2007)
  "Credit Correlation: life after copulas".
  World Scientific Publishing Company.



- D. Brigo, A. Pallavicini and R. Torresetti (2010).
  "Credit Models and the Crisis: a journey into CDOs, copulas, correlations and dynamic models".
  Wiley, Finance.

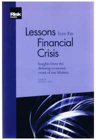


$\longrightarrow$ See also "Credit Models and the crisis or: how I learned to stop worrying and love the CDOs" (2009). Available at ssrn.com.

Reference Books – II

- Bielecki, T., Brigo, D., and Patras, F. (2011)
  "Credit Risk Frontiers: subprime crisis, pricing and hedging, CVA, MBS, ratings, and liquidity".
  Wiley Bloomberg Press, Financial Series.

- Berd, A. (2011)
  "Lessons from the Financial Crisis: insights from the defining economic event of our lifetime".
  Risk Books.



Talk Outline

1. Multi-Name Credit Modelling: the case of CDOs
   - An introduction to CDOs
   - Dynamical Loss Models
   - Recovery Dynamics and Systemic Risk
2. Counterparty Risk: pricing within Basel III framework
3. Interest-Rate Modelling: splitting of yield curves
4. Conclusions and Further Developments

How I Learned to Stop Worrying and Love the CDOs – I

- In a CDO there are two parties, a protection buyer and a protection seller.
  - $\longrightarrow$ Protection is bought (and sold) on a reference pool of M names.
  - $\longrightarrow$ Most liquid CDOs (iTraxx or CDX) consider a pool of $M = 125$ names.
- The names may default, generating losses (L) to investors exposed to those names.
  - $\longrightarrow$ Each time a name defaults the protection seller pays the protection buyer for the suffered loss.
- If the CDO is *tranched*, then only a portion of the loss of the portfolio between two percentages A and B is repayed.

$$L_t^{A,B} := \frac{M}{B-A}\left[\left(\frac{L_t}{M} - A\right) 1_{\left\{A<\frac{L_t}{M}<B\right\}} + (B-A)\, 1_{\left\{\frac{L_t}{M}>B\right\}}\right]$$

How I Learned to Stop Worrying and Love the CDOs – II

- Since tranched loss is a non-linear function of single-name losses, the tranche expectation will depend both on:
  1. marginal distributions of the single names' defaults, and on
  2. dependency (or with abuse of language "correlation") among different names' defaults.
- The complete description is either the whole multivariate distribution or the so-called *copula function* where marginal distributions have been standardized to uniform distributions.

$$F_X(x) := \mathbb{Q}\{\, X \leq x \,\} \;, \quad F_Y(y) := \mathbb{Q}\{\, Y \leq y \,\}$$

$$C(u,v) := \mathbb{Q}\left\{\, X \leq F_X^{-1}(u), Y \leq F_Y^{-1}(v) \,\right\}$$

- Notice that copulas do not define a dynamics for default processes and the choice of a particular copula family is arbitrary: Gaussian, t-Student, Archimedean, Marshall-Olkin, ...

How I Learned to Stop Worrying and Love the CDOs – III

- The dependence of the tranche on "correlation" is crucial.

  $\longrightarrow$ The market assumes a Gaussian copula connecting the defaults of the 125 names, parametrized by a correlation matrix with

  $$125 \cdot 124/2 = 7750 \quad \text{entries.}$$

  $\longrightarrow$ However, when looking at a tranche:

  $$7750 \quad \text{parameters} \quad \longrightarrow \quad 1 \quad \text{parameter.}$$

- David Li, 2005, Wall Street Journal: [...] *"The most dangerous part,"* Mr. Li himself says of the model, *"is when people believe everything coming out of it."* Investors who put too much trust in it or do not understand all its subtleties may think they have eliminated their risks when they have not.

CDOs on Different Asset Classes – I

- CDOs are available on other asset classes, such as
  - $\longrightarrow$ loans (CLO),
  - $\longrightarrow$ residential mortgage portfolios (RMBS),
  - $\longrightarrow$ commercial mortgages portfolios (CMBS), and on and on.
- For many of these CDOs, and especially RMBS, quite related to the asset class that triggered the crisis, the problem is in the data rather than in the models.
  - $\longrightarrow$ At times data for valuation in mortgages CDOs (RMBS and CDO of RMBS) can be distorted by fraud.
- Even bespoke corporate pools have no data from which to infer default "correlation" and dubious mapping methods are used.
- An intersting example: pricing a CDO on the following underlying...

CDOs on Different Asset Classes – II

- . . . a recently renovated condominium including Brazilian hardwood, granite countertops, and a value of 275,000 USD.

CDOs on Different Asset Classes – III

- At times it is not even clear what is in the portfolio, e.g. from the offering circular of a huge RMBS (more than 300.000 mortgages)

Type of property	% of Total
Detached Bungalow	2.65%
Detached House	16.16%
Flat	13.25%
Maisonette	1.53%
New Property	0.02%
Not Known	2.49%
Other	0.21%
Semi Detached Bungalow	1.45%
Semi Detached House	27.46%
Terraced House	34.78%

Beyond copulas – I

- Alternative models for implied correlations, based on different parametrizations, were proposed.
  - $\longrightarrow$ For instance the popular base correlation model, recently extended with stochastic correllation as in Amraoui and Hitier (2008).
- There are several publications that appeared pre-crisis and that questioned the Gaussian copula and implied correlations.
  - $\longrightarrow$ 2005, Wall Street Journal: How a Formula [Base correlation + Gaussian Copula] Ignited Market That Burned Some Big Investors.
- For further details see Torresetti, Brigo and Pallavicini (2006) "Implied Correlation: a paradigm to be handled with care".

Beyond copulas – II

- Brigo, Pallavicini and Torresetti (2006,2007) propose *default clustering* with the GPL and GPCL models.
- Is default clustering a realistic feature ?
  - $\longrightarrow$ Thrifts in the early 90s at the height of the loan and deposit crisis.
  - $\longrightarrow$ Airliners after 2001.
  - $\longrightarrow$ Autos and financials more recently.
- From the September, 7 2008 to the October, 8 2008, we witnessed *seven* credit events: Fannie Mae, Freddie Mac, Lehman Brothers, Washington Mutual, Landsbanki, Glitnir, Kaupþing.

Beyond copulas – III

- Errais, Giesecke and Goldberg (2006) introduce *self-excitement* effects, namely one default increases the intensity of others.
- Is self-excitement a realistic feature ?
  - ⟶ The collapse of Lehman Brothers brought the financial system near to a breakdown.
  - ⟶ Lehman was an important node within a network of derivative contracts: it sold CDS's on a large number of firms and it was itself a reference entity in many other CDS's.
  - ⟶ Its default triggered other insurance sellers into default, leaving the corresponding protection buyers with losses, etc...
- Notice that default clustering is a extreme way of modelling self-excitement.

Generalized Poisson (Cluster) Loss Model – I

- We model the total number of defaults in the pool by t as

$$Z_t := \sum_{j=1}^{n} \alpha_j Z_j(t)$$

  (for integers α_j) where Z_j are independent Poisson processes.
- This is consistent with the Common Poisson Shock framework, where defaults are linked by a Marshall-Olkin copula, see Lindskog and McNeil (2003).
- If Z_j jumps there are as many defaults as the value of α_j.
  $\longrightarrow$ Just one default (idiosyncratic) if $\alpha_j = 1$, or the whole pool in one shot (total systemic risk) if $\alpha_j = M$, otherwise for intermediate values we have defaults of whole sectors.

Generalized Poisson (Cluster) Loss Model – II

- Modelling the counting process as a sum of Poisson processes may lead to an infinite number of defaults.
- A first solution (GPL) is modifying the counting process so that it does not exceed the number of names, by simply capping Z_t to M, regardless of cluster structures:

$$C_t := \min(Z_t, M)$$

- That choice works at aggregate loss level, but it does not really go down towards single names' dynamics.

  $\longrightarrow$ The aggregate loss is capped, but we cannot track which single name is jumping.

Generalized Poisson (Cluster) Loss Model – III

- A second solution (GPCL) is forcing clusters to jump only once and deduce single names defaults consistently.
- We introduce a set of independent Poisson processes $\widetilde{N}_s$ for each cluster s, and we define the indicator J_s as given by

$$J_s(t) := \prod_{k \in s} \prod_{s' \ni k} 1_{\{\widetilde{N}_{s'}(t)=0\}}$$

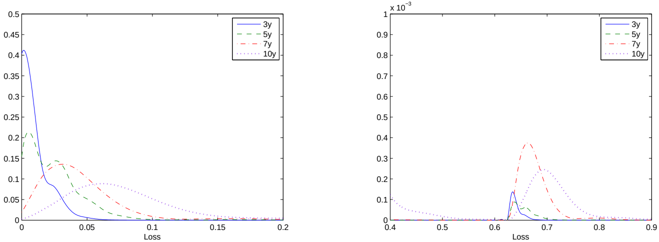
leading to the following single-name and multi-name dynamics

$$dN_k(t) = \sum_{s \ni k} J_s(t^-) d\widetilde{N}_s(t), \quad dC_t = \sum_{j=1}^{n} \alpha_j \sum_{|s|=j} J_s(t^-) d\widetilde{N}_s(t)$$

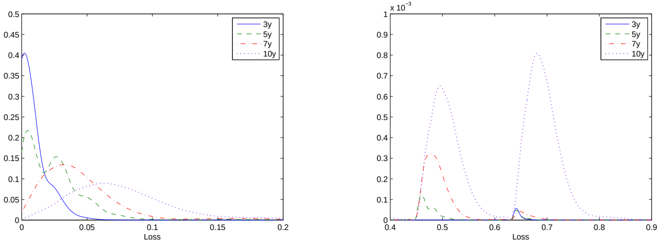
- That choice is a real top-down model, but it is combinatorially more complex.

Implied iTraxx Loss Distribution on 2-Oct-2006

GPL



GPCL



Including Systemic Risk within GPL/GPCL models – I

- The market since 2008 has been quoting CDOs with prices assuming that the super-senior tranche would be impacted to a level impossible to reach with recoveries around 40%.
  - $\longrightarrow$ Only huge losses affect super-senior tranche pricing: at least one fourth of the pool for iTraxx.
  - $\longrightarrow$ We can assign a small (or a zero) recovery to extreme events (higher modes of GPL/GPCL model).
- We assign a recovery of zero to the systemic (or Armageddon) event, corresponding to $\alpha_n = M$ mode, while we allow a recovery of $R = 40\%$ for other default events.
  - $\longrightarrow$ See Brigo, Pallavicini and Torresetti (2009,2010).
- In GPL/GPCL dynamic loss models recovery can be made a function of default rate C or portfolio loss L, see Brigo Pallavicini and Torresetti (2007) for more discussion.

Including Systemic Risk within GPL/GPCL models – II

- We introduce the stopping time $\hat{\tau}$ as the minimum time between the Armageddon jump event or the time when the reduced pool without Armageddon component has completely defaulted. This is also the time when the full pool has defaulted.

$$\hat{\tau} := \inf\left\{ t : \sum_{j=1}^{n} \alpha_j Z_j(t) \geq M \right\}$$

so that we obtain

$$C_t = M 1_{\{\hat{\tau} \leq t\}} + z_t 1_{\{\hat{\tau} > t\}} \;, \quad z_t := \sum_{j=1}^{n-1} \alpha_j Z_j(t)$$

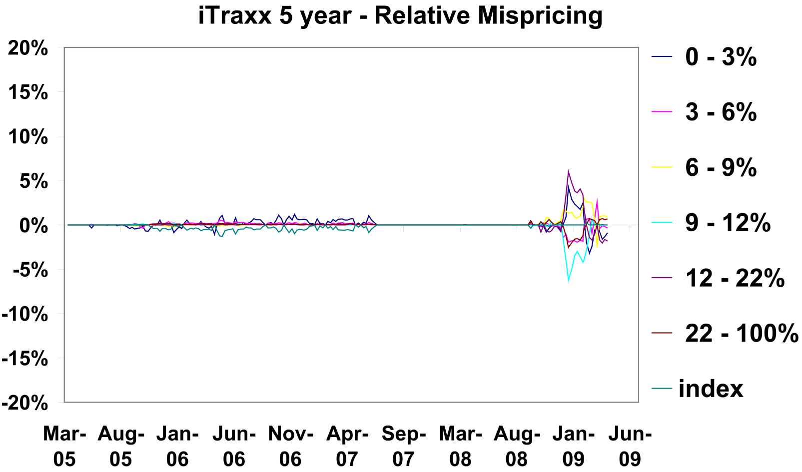
Including Systemic Risk within GPL/GPCL models – III

- Now, we assign a recovery of R to all default events but to the Armageddon event we consider with zero recovery, and after some algebra we can write the loss process as given by

$$L_t = (M - R\, z_{\hat{\tau}})1_{\{\hat{\tau}\leq t\}} + (1-R)z_t 1_{\{\hat{\tau}>t\}}$$

- Notice that L now depends on z both at full-pool exhaustion time $\hat{\tau}$ and at terminal time t.
- In dynamic loss models recovery can be made a function of the default rate C_t, see Brigo Pallavicini and Torresetti (2007).
  - $\longrightarrow$ Here, we use the above simple methodology to allow losses of the pool to penetrate beyond $1-R$ and thus affect severely even the most senior tranches, in line with market super-senior quotations.

GPL Calibration through the Crisis



Talk Outline

Bilateral Collateralized Credit Valuation Adjustment

- We define the (risk-neutral) Bilateral Collateralized Credit Valuation Adjustment (BCCVA) as

$$\mathrm{BC}_{\mathrm{CVA}}(t,T) := \mathbb{E}_t[\,\bar{\Pi}(t,T;C)\,] - \mathbb{E}_t[\,\Pi(t,T)\,]$$

  where the expectation is taken under risk-neutral measure, and
  - → $\Pi(t,T)$ is the sum of all discounted payoff terms between t and T;
  - → $\bar{\Pi}(t,T;C)$ is the same quantity subject to counterparty's default risk, and mitigated by collateral margining;
  - → C_t is the collateral account used by the margining procedure;
- We introduce both the counterparty's and the investor's time of default τ_C and τ_I, and the (first) default time as $\tau := \min\{\tau_C, \tau_I\}$.
- Funding costs can be added as an additional pricing term compensating for the costs needed to complete each cash-flow transaction.

Mitigating Counterparty Credit Risk – I

- The ISDA Master Agreement lists two different tools to reduce exposure:
  - → margining with collaterals, namely the right of recourse to some asset of value that can be sold or the value of which can be applied in the event of default on the transaction; and
  - → close-out netting rules, which state that if a default occurs, multiple obligations between two parties are consolidated into a single net obligation.
- When we monitor a (symmetric) risk in a bilateral agreement, we should introduce a "metric" which is shared by both parties.
  - → The ISDA Master Agreement defines the term *exposure* to be the netted mid-market mark-to-market value of the transaction.
- We name such (pre-default) exposure as ε_t.

$$\varepsilon_t := \mathbb{E}_t[\,\Pi(t, T)\,]$$

Mitigating Counterparty Credit Risk – II

- We consider collaterals to be posted on a Collateral Account held by a Collateral Taker, and we name its value at time t with C_t.
  - $\rightarrow$ If at time t the investor post some collateral we consider that $dC_t < 0$, the other way round if the counterparty is posting.
- In general, margining practice consists in a pre-fixed set of dates $\{t_0, \ldots, t_n\}$ during the life of a deal when both parties post or withdraw collaterals, according to their current exposure, to or from an account held by the Collateral Taker.
  - $\rightarrow$ The account is usually remunerated at over-night rate
  - $\rightarrow$ We name $D(t, T)$ the over-night (risk-free) discount factor.
- A simple margining rule is the following

$$C_{t_0} = C_{t_n} = 0 \,, \quad C_{t_i} = \varepsilon_{t_i} \,, \quad C_u = \frac{\varepsilon_{\beta(u)}}{D(\beta(u), u)}$$

where $t_0 < u < t_N$ and $\beta(u)$ is the last update time not after u.

Netting Rules – I

- In case of default of one party, the surviving party should evaluate the transactions just terminated, due to the default event occurrence, to claim for a reimbursement after the application of netting rules to consolidate the transactions, inclusive of collateral accounts.
- The ISDA Master Agreement defines the term *close-out amount* to be the amount of the losses or costs of the surviving party would incur in replacing or in providing for an economic equivalent, by acting in good faith and by using commercially reasonable procedures.
- We name the close-out amount priced at time t by the investor on counterparty's default with $\varepsilon_{I,t}$ (and $\varepsilon_{C,t}$ in the other case, namely when the investor is defaulting).
  - $\rightarrow$ The close-out amount is not a symmetric quantity w.r.t. the exchange of the rôle of two parties.

Netting Rules – II

- Close-out amounts are calculated after a default event happens, while exposures depened on the margining procedure which takes place before any default event. See Brigo, Capponi and Pallavicini (2011) or Fujii and Takahashi (2011).
  - $\rightarrow$ Expectations required by exposure calculation:

$$1_{\{u<\tau_C\}}1_{\{u<\tau_I\}}\mathbb{E}_u[\,\Pi(u,T)\,]$$

  - $\rightarrow$ Expectations required by close-out amount calculation:

$$1_{\{u<\tau_I\}}\mathbb{E}_u[\,\Pi(u,T)|\{u=\tau_C\}\,]$$

- Weeber (2009) and Parker (2009) show that close-out amounts rely on the credit-worthiness of the surviving party, but also on many other factors.
  - $\rightarrow$ For instance, the costs of terminating, liquidating or re-establishing any hedge or related trading position, and the costs of funding.

Cash Flows on Default Event

- We consider all the situations may arise on counterparty's or investor's default event (R_C and R_I are recovery rates).

$$
\begin{aligned}
\bar{\Pi}(t, T; C) = {} & 1_{\{\tau > T\}} \Pi(t, T) + 1_{\{\tau < T\}} (\Pi(t, \tau) + D(t, \tau) C_{\tau^-}) \\
& + 1_{\{\tau = \tau_C < T\}} D(t, \tau) 1_{\{\varepsilon_{I,\tau} < 0\}} 1_{\{C_{\tau^-} > 0\}} (\varepsilon_{I,\tau} - C_{\tau^-}) \\
& + 1_{\{\tau = \tau_C < T\}} D(t, \tau) 1_{\{\varepsilon_{I,\tau} < 0\}} 1_{\{C_{\tau^-} < 0\}} ((\varepsilon_{I,\tau} - C_{\tau^-})^- + (\varepsilon_{I,\tau} - C_{\tau^-})^+) \\
& + 1_{\{\tau = \tau_C < T\}} D(t, \tau) 1_{\{\varepsilon_{I,\tau} > 0\}} 1_{\{C_{\tau^-} > 0\}} ((\varepsilon_{I,\tau} - C_{\tau^-})^- + R_C (\varepsilon_{I,\tau} - C_{\tau^-})^+) \\
& + 1_{\{\tau = \tau_C < T\}} D(t, \tau) 1_{\{\varepsilon_{I,\tau} > 0\}} 1_{\{C_{\tau^-} < 0\}} (R_C \varepsilon_{I,\tau} - C_{\tau^-}) \\
& + 1_{\{\tau = \tau_I < T\}} D(t, \tau) 1_{\{\varepsilon_{C,\tau} > 0\}} 1_{\{C_{\tau^-} < 0\}} (\varepsilon_{C,\tau} - C_{\tau^-}) \\
& + 1_{\{\tau = \tau_I < T\}} D(t, \tau) 1_{\{\varepsilon_{C,\tau} > 0\}} 1_{\{C_{\tau^-} > 0\}} ((\varepsilon_{C,\tau} - C_{\tau^-})^+ + (\varepsilon_{C,\tau} - C_{\tau^-})^-) \\
& + 1_{\{\tau = \tau_I < T\}} D(t, \tau) 1_{\{\varepsilon_{C,\tau} < 0\}} 1_{\{C_{\tau^-} < 0\}} ((\varepsilon_{C,\tau} - C_{\tau^-})^+ + R_I (\varepsilon_{C,\tau} - C_{\tau^-})^-) \\
& + 1_{\{\tau = \tau_I < T\}} D(t, \tau) 1_{\{\varepsilon_{C,\tau} < 0\}} 1_{\{C_{\tau^-} > 0\}} (R_I \varepsilon_{C,\tau} - C_{\tau^-})
\end{aligned}
$$

BCCVA Master Formula

- We obtain, after a little of algebra, the general expression for collateralized bilateral CVA ($\text{L}_{\text{GD}_k} := 1 - R_k$ with $k \in \{C, I\}$).
  → See Brigo, Capponi, Pallavicini and Papatheodorou (2011).

$$\text{BC}_{\text{CVA}}(t, T; C) = \text{C}_{\text{AM}}(t, T) - \text{C}_{\text{CVA}}(t, T; C) + \text{C}_{\text{DVA}}(t, T; C)$$

with

$$\text{C}_{\text{AM}}(t, T) := -\mathbb{E}_t\left[1_{\{\tau < T\}} D(t, \tau) \left(\varepsilon_\tau - 1_{\{\tau = \tau_C\}} \varepsilon_{I,\tau} - 1_{\{\tau = \tau_I\}} \varepsilon_{C,\tau} \right) \right]$$

and

$$\text{C}_{\text{CVA}}(t, T; C) := +\mathbb{E}_t\left[1_{\{\tau = \tau_C < T\}} D(t, \tau) \text{L}_{\text{GD}_C} (\varepsilon^+_{I,\tau} - C^+_{\tau^-})^+ \right]$$

$$\text{C}_{\text{DVA}}(t, T; C) := -\mathbb{E}_t\left[1_{\{\tau = \tau_I < T\}} D(t, \tau) \text{L}_{\text{GD}_I} (\varepsilon^-_{C,\tau} - C^-_{\tau^-})^- \right]$$

Formulae for Collateralized Bilateral CVA – I

- We need a recipe to calculate close-out amounts ε_{I,τ_C} and ε_{C,τ_I}, that, in the practice, are approximated from today exposure corrected for haircuts or add-ons.
  - → We could also approximate close-out amounts with the value of a replacement operation, as in Brigo, Capponi, Pallavicini and Papatheodorou (2011).
- As a first case we consider all the exposures being evaluated at mid-market, while after this section we show a possible way to relax such approximation by introducing nested BCCVA approximation.

$$\varepsilon_{I,t} \doteq \varepsilon_{C,t} \doteq \varepsilon_t$$

- Thus, in such case we obtain for collateralized bilateral CVA

$$\begin{aligned}\mathrm{BC}\textsc{cva}(t,T;C) = &-\mathbb{E}_t\left[\,1_{\{\tau=\tau_C<T\}} D(t,\tau) \mathrm{L}\textsc{gd}_C (\varepsilon_\tau^+ - C_{\tau^-}^+)^+\,\right] \\ &- \mathbb{E}_t\left[\,1_{\{\tau=\tau_I<T\}} D(t,\tau) \mathrm{L}\textsc{gd}_I (\varepsilon_\tau^- - C_{\tau^-}^-)^-\,\right]\end{aligned}$$

Formulae for Collateralized Bilateral CVA – II

- If we remove collateralization ($C_t = 0$), we recover the result of Brigo and Capponi (2008), and used in Brigo, Pallavicini and Papatheodorou (2009).

$$\begin{aligned}\mathrm{B}\text{CVA}(t,T) &:= \mathrm{BC}\text{CVA}(t,T;0)\\ &= -\,\mathbb{E}_t\left[\,1_{\{\tau=\tau_C<T\}}D(t,\tau)\mathrm{L}\text{GD}_C\varepsilon_\tau^+\,\right]\\ &\quad -\,\mathbb{E}_t\left[\,1_{\{\tau=\tau_I<T\}}D(t,\tau)\mathrm{L}\text{GD}_I\varepsilon_\tau^-\,\right]\end{aligned}$$

- If we remove collateralization ($C_t = 0$) and we consider a risk-free investor ($\tau_I \to \infty$), we recover the result of Brigo and Pallavicini (2007), but see also Canabarro and Duffie (2004).

$$\begin{aligned}\mathrm{U}\text{CVA}(t,T) &:= \mathrm{B}\text{CVA}(t,T)|_{\tau_I\to\infty}\\ &= -\,\mathbb{E}_t\left[\,1_{\{\tau_C<T\}}D(t,\tau_C)\mathrm{L}\text{GD}_C\varepsilon_{\tau_C}^+\,\right]\end{aligned}$$

Wrong- and Right-Way Risk – I

- A working paper issued by the Basel Committee in December 2009 named *"Strengthening the resilience of the banking sector"* investigates an extension of the Basel II framework to deal with the market transformations due to the credit-crunch crisis.
- The Basel Committee in December 2010 issued another paper named *"Basel III: A global regulatory framework for more resilient banks and banking systems"* containing the guidelines for counterparty risk management.
- Particularly stressed is the dependence between market and credit risks, also known as wrong- and right-way risk, which was not adequately incorporated into the Basel II framework.

Wrong- and Right-Way Risk – II

- Basel II framework defines the supervisory scaling parameter [Annex IV, 69], which describes in a synthetic way the impact of the dependency between market and credit risk.

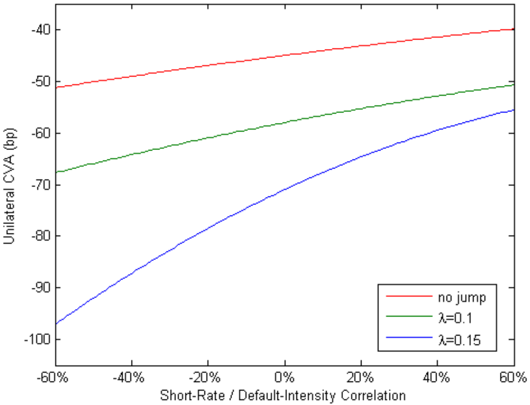
$$\beta := \frac{\mathrm{U_{CVA}}}{\mathrm{U_{CVA}}^0}$$

  where $\mathrm{U_{CVA}}$ and $\mathrm{U_{CVA}}^0$ are respectively the unilateral credit valuation adjustment considering and not considering correlations.

- Notice that β is usually set equal to 1.4 [Annex IV, 90], while, for instance, in Brigo and Pallavicini (2007) values greater then 1.4 are found for relevant portfolio's examples.

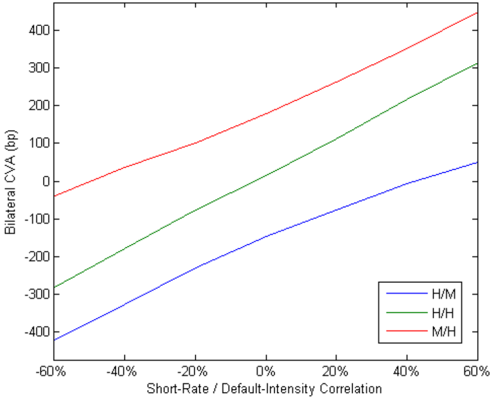
Wrong- and Right-Way Risk – III

- Unilateral credit valuation adjustment for an at-the-money IRS with ten year maturity and one year coupon tenor for different credit-spread models. See Brigo and Pallavicini (2007).



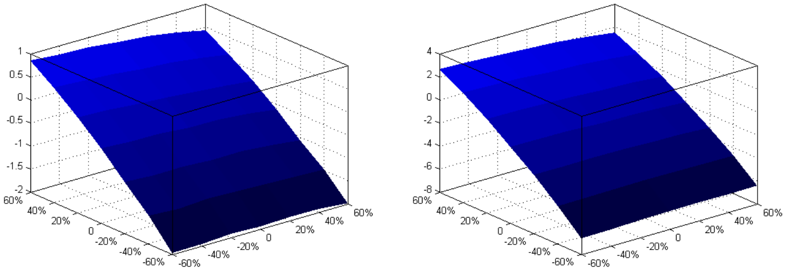
Wrong- and Right-Way Risk – IV

- Bilateral CVA for a netted portfolio of IRS with ten year and one year coupon tenor maturity for different creditworthiness of counterparty and investor. See Brigo, Pallavicini and Papatheodorou (2009).



Wrong- and Right-Way Risk – V

- Collateralized bilateral CVA for an IRS with ten year maturity and one year coupon tenor with different choices of interest-rate/credit-spread correlation (left-side axis) and default-time correlation (right-side axis) with collateral update intervals of one week (left panel) and three months (right panel).



Central Counterparties and Systemic Risk – I

- The recent credit crisis has has emphasized the importance of *contagion* and *systemic risk*, defined as the risk that interconnected financial institution can undermine the stability of financial system as a whole.
  - → See the case of CDO's where we consider models with default probability clusters corresponding to joint default of a large number of entities (sectors) of the economy.
  - → Carmona and Crèpey (2010) propose imporatance-sampling techniques to deal with extreme events and contagions with Monte Carlo simulations.
- Regulators could not anticipated the impact of defaults partly due to the lack of relevant indicators describing the interaction of the different components of financial system.
- Development of a dynamic and interacting credit risk system may provide better indicators of systemic risk, and help predicting the onset of a crisis.

Central Counterparties and Systemic Risk – II

- Cont, Minca and Moussa (2009) propose a network approach to contagion modeling, where nodes of a random graph are financial institutions (banks, funds), and edges represent counterparty exposures.
  - → The effect of a central counterparty can be modeled by adding a node to the CDS network and redirecting all CDS contracts into this node.
  - → Central clearing-houses should be properly designed to effectively reduce systemic risk.
  - → Systemic risk depends on network properties and it may have little correlation with conventional risk measures.

Central Counterparties and Systemic Risk – III

- Dai Pra, Runggaldier, Sartori and Tolotti (2009) develop a dynamic contagion model via interacting particle systems, which describes the propagation of financial distress in a network of defaultable financial market participants.
  - → Using limiting arguments, they employ large deviation theory to quantify losses incurred by a bank in large credit portfolios.
  - → The model allows to explain default clustering and to view a credit-crisis event as a microeconomic phenomenon driven by endogenous financial indicators.

Talk Outline

Interest-Rate Modelling – I

- Classical interest-rate models were formulated to satisfy *by construction* no-arbitrage relationships, which allow to hedge forward-rate agreements in terms of zero-coupon bonds.
- For instance, these models predict forward rates of different tenors to be related to each other by strong constraints via zero-coupon bond prices. Indeed, for three times $T_0 < T_1 < T_2$ we get

$$F_t(T_0, T_1) = \frac{1}{T_1 - T_0}\left(\frac{P_t(T_0)}{P_t(T_1)} - 1\right)$$

- In practice, these no-arbitrage relationships might not hold.
  $\longrightarrow$ An example is provided by basis-swap spread quotes, which are significantly non-zero, while they should be equal to zero if such constraints held.

Interest-Rate Modelling – II

- Basis-swap spread for six-months vs. three-months Euribor rates on a swap with maturity one year.

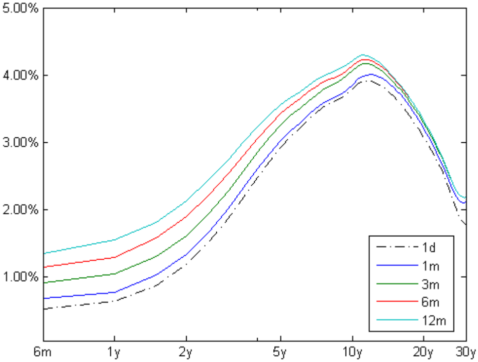
Interest-Rate Modelling – III

- Starting from summer 2007, with the spreading of the credit crunch, market quotes of forward rates and zero-coupon bonds began to violate the usual no-arbitrage relationships under
  - $\longrightarrow$ the pressure of a liquidity crisis reducing the credit lines, and
  - $\longrightarrow$ the possibility of a systemic break-down suggesting that counterparty risk could not be considered negligible any more.
- The resulting picture, see Henrard (2007), describes a money market where each forward rate seems to act as a different underlying assets.

$$F_t(T_0, T_1) \quad \rightsquigarrow \quad \begin{cases} F_t^{1\mathrm{m}}(T_0, T_0 + 1\mathrm{m}) \\ F_t^{3\mathrm{m}}(T_0, T_0 + 3\mathrm{m}) \\ F_t^{6\mathrm{m}}(T_0, T_0 + 6\mathrm{m}) \\ F_t^{12\mathrm{m}}(T_0, T_0 + 12\mathrm{m}) \end{cases}$$

Interest-Rate Modelling – IV

- Forward rates in a multi-curve framework. On the x-axis we have the rate start-dates expressed in years, while on the y-axis we have the value of the forward rates. Market data observed on 14 June 2010. See Pallavicini and Tarenghi (2010).



Counterparty vs. Liquidity Risk – I

- A closer look at the Euro money market makes clear that quoted instruments are indexed on three reference indices.
  1. Eonia is an *effective* rate calculated from the weighted average of all overnight unsecured lending transactions undertaken in the interbank market.
  2. Euribors are *offered* rates at which Euro interbank term deposits of different maturities are traded by one prime bank to another one.
  3. Eurepos are *offered* rates at which Euro interbank secured money market transactions are traded.
- Eonia and Euribor rates are unsecured, so that they incorporate the default risk of the counterparty of the transaction, while Eurepo rates are secured and free of credit risk.

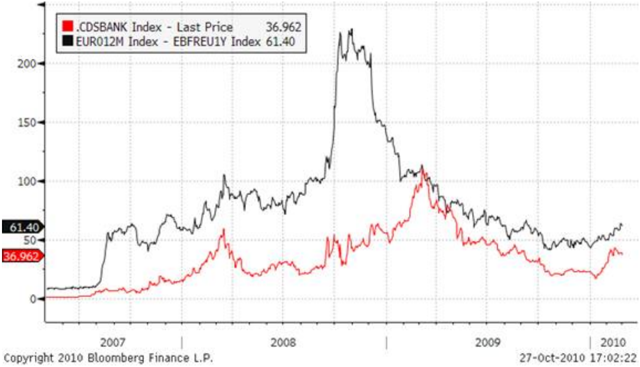
$\longrightarrow$ Eurepo rates could seem the natural proxy for risk-free rates.

Counterparty vs. Liquidity Risk – II

- There are many empirical studies supporting the idea that Euribor rate levels cannot be utterly justified by counterparty credit risk arguments.
  - $\longrightarrow$ See, for instance, the European Central Bank working paper by Eisenschmidt and Tapking (2009), or the contribution by Heider et al. (2009).
- There is evidence of a large, persistent and time varying component of the Euribor-Eurepo spread that cannot not be explained only by counterparty credit risk.
  - $\longrightarrow$ The sharp rise in the Euribor-Eurepo spread of September 2008 is only found three-four months later in the CDS spread series, confirming that a liquidity crisis needs time to evolve as credit crisis.

Counterparty vs. Liquidity Risk – III

- Historical series of Euribor-1y minus Eurepo-1y spread (black line) and a synthetic index formed by senior one-year CDS of a basket of twelve representative European banks (red line). Values are in basis points.



Multiple-Curve Modelling – I

- We wish to introduce a *parsimonious* model which is able to describe a multi-curve setting by starting from a limited number of (Markov) processes.
- Our proposal is to extend the logic of the HJM framework to describe with a single family of Markov processes all the yield curves we are interested in.
- In the literature other authors proposed generalizations of the HJM framework, see for instance Carmona (2004), Andreasen (2006), or Chiarella (2010).

  $\longrightarrow$ In particular, in recent papers Martìnez (2009) and Fujii et al. (2010) extended the HJM framework to incorporate multiple-yield curves and to deal with foreign currencies.

Multiple-Curve Modelling – II

- Let us summarize the basic requirements the model must fulfill:
  1. existence of a risk-free curve, with instantaneous forward rates $f_t(T)$
  2. existence of Euribor rates, typical underlying of traded derivatives, with associated forward rates $F_t(T,x)$
  3. no-arbitrage dynamics of the $f_t(T)$ and the $F_t(T,x)$ (both being T-forward measure martingales) ensuring the limit case

$$f_t(T) = \lim_{x\to 0} F_t(T,x)$$

  4. possibility of writing both the $f_t(T)$ and the $F_t(T,x)$ as functions of a common family of Markov processes.
- While the first two requisites are related to the set of financial quantities we are about to model, the last two are conditions we impose on their dynamics, and will be granted by a befitting choice of model volatilities.

Generalized HJM Dynamics – I

- We choose, under the risk-free T-forward measure, the following dynamics.

$$df_t(T) = \sigma_t^*(T) \cdot dW_t$$

$$\frac{d(\kappa(T,x) + F_t(T,x))}{\kappa(T,x) + F_t(T,x)} = \Sigma_t^*(T,x) \cdot dW_t$$

with $f_0(T)$ and $F_0(T,x)$ bootstrapped from market quotes, and

$$\sigma_t(T) := \sigma_t(T;T,0)\ , \quad \Sigma_t(T,x) := \int_{T-x}^{T} du\, \sigma_t(u;T,x)$$

where $\sigma_t(u;T,x)$ is a (row) volatility vector process, and W_t is a (row) vector of independent Brownian motions.

- The set of shifts $\kappa(T,x)$ must satisfy: $\kappa(T,x) \approx 1/x$ if $x \approx 0$.

Generalized HJM Dynamics – II

- Let us analyse more in detail the dynamics of the shifted forward rates under risk-neutral measure. By integrating the SDE over the time period $[0, t]$ we get

$$\ln\left(\frac{\kappa(T,x)+F_t(T,x)}{\kappa(T,x)+F_0(T,x)}\right) = \int_0^t \Sigma_s^*(T,x)\cdot\left(dW_s - \frac{1}{2}\Sigma_s(T,x)\,ds + \int_s^T du\,\sigma_s(u;u,0)\,ds\right)$$

- Our goal is substituting the right-hand side with a tractable formula expressed in term of a family of Markov processes.

  $\longrightarrow$ This is a requirement similar to the one adopted in the HJM framework when zero-coupon bond pricing formula is considered.

Generalized HJM Dynamics – III

- To ensure the tractability and a Markovian specification of the model, we extend the single-curve HJM approach of Ritchken and Sankarasubramanian (1995), by setting

$$\sigma_t(u; T, x) := h_t \cdot (q(u; T, x) g(t, u))$$

where h_t is a matrix adapted process, q is a deterministic vector function, and g is defined as

$$g(t, u) := \exp\left\{ - \int_t^u dv \, \lambda(v) \right\}$$

with λ a deterministic vector function.

- Further, We add the condition $q(u; u, 0) = 1$ to ensure that in the limit case $x \to 0$ we recover the standard HJM separability condition.

Emerging Driving Markov Processes – I

- Hence, by plugging the expression for the volatility, we get

$$\ln\left(\frac{\kappa(T,x)+F_t(T,x)}{\kappa(T,x)+F_0(T,x)}\right) =$$

$$G^*(t, T-x, T; T,x)\cdot\left(X_t + Y_t\cdot\left(G_0(t,t,T) - \frac{1}{2}G(t, T-x, T; T,x)\right)\right)$$

where the vectorial deterministic functions G_0 and G are defined as

$$G_0(t, T_0, T_1) := \int_{T_0}^{T_1} dv\, g(t,v)$$

$$G(t, T_0, T_1; T,x) := \int_{T_0}^{T_1} dv\, q(v; T,x) g(t,v)$$

Emerging Driving Markov Processes – II

- The vector process X_t and the matrix process Y_t are defined as

$$X_t := \sum_{k=1}^{N} \int_0^t (h_s g(s,t))^* \cdot \left(dW_s + h_s \int_s^t dv\, g(s,v)\, ds \right)$$

$$Y_t := \int_0^t ds\, (h_s g(s,t))^* \cdot (h_s g(s,t))$$

- Thanks to our volatility assumption they result to be Markov, and their dynamics is given by

$$dX_t = (Y_t^* \cdot 1 - \lambda(t) X_t)\, dt + h_t^* \cdot dW_t$$

$$dY_t = (h_t^* \cdot h_t - (\lambda^*(t) Y_t + Y_t \lambda(t)))\ dt$$

with $X_0 = 0,\ Y_0 = 0.$

Implying Volatilities for All Rate Tenors – I

- The reconstruction formula for forward Euribor rates is the analogous of standard HJM formula for zero-coupon bonds and it is our main result.
  - $\longrightarrow$ This important feature is consistent with the requirement of a model capable to *directly* describe market relevant quantities.
- Further, it is possible to check a posteriori that our reconstruction formula has as limit case for $x \to 0$ the HJM formula for zero-coupon bonds.
- As an example, we consider a particular model specification within the extended HJM framework, and we calibrate it to at-the-money swaption and cap quotes.

Implying Volatilities for All Rate Tenors – II

- We consider the volatility process h_t to be in the form

$$h_t := \varepsilon(t) h R \,, \quad \rho := R^* R$$

where h is a constant vector, R is an upper triangular matrix, and we allow for a time varying common volatility shape

$$\varepsilon(t) := 1 + (\beta_0 - 1 + \beta_1 t) e^{-\beta_2 t}$$

where β_0, β_1, β_2 are three positive constants.

- As for the tenor-maturity factors q and κ, we chose a maturity independent form of the type

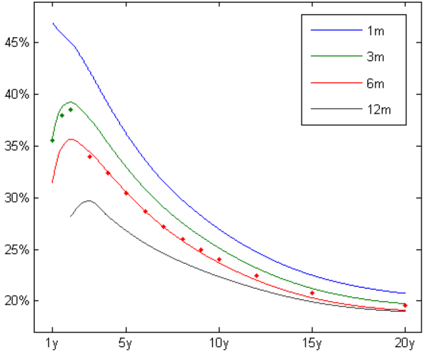
$$q(u; T, x) := \exp\{-x\eta\} \,, \quad \kappa(T, x) := 1/x$$

- Numerical tests are done with a mixture with weight w of two scenarios ($j \in \{1, 2\}$) both with two driving factors ($i \in \{1, 2\}$):

$$\left\{ \lambda_i^j, h_i^j, \eta_i^j; \rho_{12}^j; \beta_0^j, \beta_1^j, \beta_2^j; \right\}$$

Implying Volatilities for All Rate Tenors – III

- At-the-money cap volatilities implied by the mixture model for 1m, 3m, 6m and 12m rate tenors. Market quotes as big dots. On the *x*-axis we have the cap maturity, while on the *y*-axis we have the implied volatility. Market data observed on 15 February 2011.



Talk Outline

Conclusions

- We considered the impact of the credit crisis on pricing models. In particular we analyse the credit and the interest-rate markets.
- We discussed the following features:
  - $\longrightarrow$ impact of systemic risk (super-senior tranche premium, contagion effects of credit risk, splitting of yield curves);
  - $\longrightarrow$ ubiquity of credit risk (pricing must include CVA, collateral margining, netting rules,...);
  - $\longrightarrow$ complexity of market dynamics (interplay between liquidity and credit risk, risk-free vs. collateral discounting).

Further Developments – I

- Credit derivatives:
  - $\longrightarrow$ top-down portfolio loss models with reduced dimensionality consistent with single-name dynamics;
  - $\longrightarrow$ pricing models with credit contagion for a network counterparties inclusive of central clearing-houses;
  - $\longrightarrow$ modelling market/credit correlations and their calibration to market data;
  - $\longrightarrow$ porting CVA, collateral margining, netting rules, close-out amount evaluation, funding costs from the bank book to the trading book.

Further Developments – II

- Interest-rate derivatives:
  - $\longrightarrow$ interest-rate modelling under collateral measure, namely one discounting curve for each dervative contract;
  - $\longrightarrow$ investigating the multiple-curve framework to select a realistic intra-tenor dynamics;
  - $\longrightarrow$ modelling central-bank vs. market rates including monetary policies;
  - $\longrightarrow$ is any more possible modelling interest-rates without credit and liquidity effects ?

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