Description of the Code from: *Assessing Systemic Risk Due to Fire Sales Spillover Through Maximum Entropy Network Reconstruction*

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What follows is a description of the code, available at: `http://mathfinance.sns.it/network_reconstruction/`, designed to estimate and sample different ensembles of bipartite networks. In Di Gangi et al. (2015) the ensembles are used to asses the systemic risk metrics of the *Vulnerable Banks* framework described in Greenwood et al. (2015), from partial information about the bipartite weighted network structure of common asset holdings.

The matlab package can be employed also in contexts different from systemic risk assessment, to estimate and sample (bipartite) network ensembles. The reader interested in this application should look directly at 2.2

Matlab's optimization toolbox is required.

1 The *Vulnerable Banks* Framework

The framework, referred to as *Vulnerable Banks*, describes a set of N banks (more generally asset holders, that are expected to target a fixed value of their leverage) and a set of K asset classes, and assumes a simple dynamic for banks as consequence of an exogenous shock on a set of asset classes.

Bank n initially holds assets in k for value X_{nk}, and is endowed with equity E_n. Thus the total asset value of bank n is $A_n = \sum_k X_{nk}$ and the initial weight of asset class k in its portfolio is $M_{nk} = \frac{X_{nk}}{A_n}$. To each asset class is associated a parameter called market illiquidity ℓ_k, with $k = 1, ..., K$, defined as the deprecation of the class if an amount of one dollar of that class is exchanged. Besides, each bank n is provided with a total equity E_n.

Systemic risk measures are derived assuming that each asset class is hit by a negative shock $-\varepsilon_k$ with $k = 1, ..., K$. Denote by $-\boldsymbol{\varepsilon} = (-\varepsilon_1, ..., -\varepsilon_K)$ the vector whose components are the assets' shocks. The metrics defined in Greenwood et al. (2015) are

$$\mathrm{AV}\left(\mathbf{X}\right) \equiv \frac{\sum_m \left(\sum_k X_{mk}\ell_k \sum_n \left(B_n - 1\right) M_{nk} \sum_{k'} X_{nk'}\epsilon_{k'}\right)}{\sum_{n'} E_{n'}}. \tag{1}$$

$$S_n\left(\mathbf{X}\right) = \frac{\sum_m \left(\sum_k X_{mk}\ell_k \quad \left(B_n - 1\right) M_{nk} \sum_{k'} X_{nk'}\epsilon_{k'}\right)}{\sum_{n'} E_{n'}}, \tag{2}$$

$$\mathrm{IV}_n\left(\mathbf{X}\right) = \frac{\left(\sum_k X_{mk}\ell_k \sum_n \left(B_n - 1\right) M_{nk} \sum_{k'} X_{nk'}\epsilon_{k'}\right)}{E_n}. \tag{3}$$

1.1 Systemic Risk Reconstruction from Partial Information

The data needed in the *Vulnerable Banks* framework is:

- X_{nk}
- The equity of each investor E_n
- Reasonable assumptions for the illiquidity parameters of each asset class

The main purpose of our code is to allow the computation of IV AV S when the network structure X_{nk} is not exactly known, but some information about it is available, e.g. A_n andC_k are known.

The available information can be employed to define a network model that need to be estimated and are used to reconstruct systemic risk[1].

Network Models Examples of the network models available are[2]

- BIPWCM -**The information needed is** (A, C) - Described in details in Di Gangi et al. (2015), it is defined via the maximum entropy principle (see Park and Newman, 2004) and the knowledge of the strength sequences.

- MECAPM -**The information needed is** (A, C) - Described in details in Di Gangi et al. (2015), it is defined as a combination of the maximum entropy principle and the Capital Asset Pricing Model (CAPM).

[1] For a throughout description of the reconstruction see Di Gangi et al. (2015)

[2] For a complete list type the: Max_Entr_Nets('LIST',0) . The description of each model is available in the corresponding file.

2 Code Description

- Vulnerable_Banks : the only one needed for the purpose of systemic risk assessment. It computes the *Vulnerable Banks* statistics either exactly (if X_{nk} is known) or estimated from some partial information. The kind of information that can be used to reconstruct systemic risk is defined by the network model used.
- Max_Entr_Nets : Estimates and samples a network model of choice given the input statistics that define the model. It contains the code regarding network ensembles. Is called by Vulnerable_Banks, but can be used as a standalone function.

2.1 [AV,SYS,VUL] = Vulnerable_Banks(mode,input_data,equity,shock,liq)

With this function the user is able to reproduce the results of Di Gangi et al. (2015) on different data.

INPUTS

- **mode** : Indicates whether the function is required to compute the real statistics or estimate them from partial information. Possible values:
  - mode = 'REAL'; Return the systemic risk measures based on the knowledge of the weighted adjacency matrix X, expected as input_data. As a convention each row of the matrix is associated to an investor, while each column to an asset.
  - mode = 'ESTIMATE'; Estimate the vulnerable banks statistics from partial information. Using the MECAPM model.
  - mode = 'ESTIMATE-LIST'; Print a list of the models available for the estimation from partial information.
  - mode = 'ESTIMATE-BIP***'; Substituting *** with the name of a model available, i.e. 'ESTIMATE-BIPWCM' returns the estimates based on the corresponding network ensemble. The input_data needs to be exactly that required from the particular model BIP***.
- **input_data** Contains the available information about the configuration of the network structure. This info can be complete or partial. Possible values:

- When mode == 'REAL' then input_data = X; where X is the weighted adjacency matrix, that describes the bipartite weighted network of investors and assets.
- When mode == 'ESTIMATE' , input_data needs to be a cell array with 2 columns. the fist cell element needs to contain a column array with the investors' capitalizaiton, the second cell element a column of the assets capitalization.
- When mode == 'ESTIMATE-BIP***' the input data needs to contain the information required by the model BIP*** . Type help BIP*** for the requirements of a specific model BIP***.

- equity : Contains a vector of the equities of each investor. The order needs to be the same of input_data, i.e. equity(1) is the equity of the investor associated to the first row of the adjacency matrix.
- shock : If provided as an input, shock needs to be a real number between 0 and 1 that indicates the entity of the initial shock that triggers the vulnerable banks dynamics. **If not provided a shock of 0.01 is assumed**, i.e. a 1% depreciation of al the assets present in the system.
- liq : The vector of illiquidities associated to each asset. **If not provided the first asset is assumed to be cash, while all other assets have an assumed liquidity of** 10^{-13}, means that "10 billion $ of trading imbalnces lead to a price change of basis points." Note that if the unit measure is not 1$ the liquidity needs to be rescaled.

```matlab
clc
close all
clear

% Load the functions into matlab's path
addpath(genpath('/Max_entr_net_ensembles/'));

% load the data
load ../data/Data_nets.mat

% In this example we assume to known the complete
    network structure
X = Data_nets.net;

%% Example computation real values
```

```

% load the equities
equity = Data_nets.equity;

[AV_real,SYS_real,VUL_real] = Vulnerable_Banks('REAL',X,equity);

%% Example Estimation of systemic risk metrics from partial information

% In this example we use only the partial information, but  we compute it from matrix X, that is presumed to be not available when estimation is required. We do that to clearify the structure of in_data

 indata = cell(1,2);
 % the first element of the cell array is a column array
 indata{1,1} = sum(X,2);
 % the second element of the cell array is a row array
 indata{1,2} = sum(X);

[AV_est,SYS_est,VUL_est] = Vulnerable_Banks('ESTIMATE',in_data,equity);
```

2.2 [Out_MENE] = Max_Entr_Nets(model,in_data)

This function can be used if the user is interested in the estimation and sampling of the network ensembles available. A possible application is the computation of the average of a given function (not necessarily related to systemic risk) over the ensemble for the reconstruction of arbitrary network statistics from limited information, or for comparison purposes, i.e. employing the ensemble as a null model. This kind of applications have been well explored in the literature (for the definitions and an application see Park and Newman, 2004; Mastrandrea et al., 2014)

INPUTS

- **model**: Select the network model, i.e. the probability distribution over the set of graphs with given number of nodes [3] In our framework the models is defined via entropy maximization constrained by the requirement that a set of statistics has fixed values on average over the ensemble. To print a list of the available models type Max_Entr_Nets('LIST',0).

- **in_data**: A cell type array containing the values of all the statistics fixed in a given model. If the adjacency matrix is available this cell can be generated using the function indata_from_matrix_Nets(model,X).
- optional input **precision**: fixes the required precision, i.e. the maximum relative error between each observed value of the "soft constraints" and the analytically computed averages over the ensemble. The estimation stops when the required precision is reached. Default value is 10^-2.

OUTPUTS

- Out_MENE: this (Max Entropy Network Ensemble) structure has fields that describe the estimated model and allow sampling as shown in the following example

```
% if the real matrix is not available, the values
% of the required statistics (those that define
% the network model), need to be organized
% in the in_data cell array following the
% criteria used in the the function
% indata_from_matrix_Nets.
% E.g. in the BIPWCM, being sr the sum of the
% values in each row of the matrix and sc the
% sum of the columns: in_data = {sr,sc};

%If instead the real matrix X_real is available
in_data = indata_from_matrix_Nets('BIPWCM',X_real);

% estimate the model
[Out_MENE, details] = Max_Entr_Nets('BIPWCM',in_data
    ,10^-3);

% Returns the set of parameters estimated. They
```

[3]Sometimes referred to as grand-canonical ensemble, in the sense that the total number of links is not restricted, but constraints are enforced on average.

```
% are organized in a unique N + M column vector
% whose first N components are the parameters
% associated to the row sums and the last M
% are associated to the column  sums.
Out_MENE.parameters

% returns the expected matrix. Is a function
% that computes the matrix from the parameters
% stored in the parameters field
Out_MENE.exp_matrix()

% samples 100 matrices from the BIPWCM ensemble,
% and returns a cell array containing the
% different realizations
sample = Out_MENE.sample(100);
```

References

Di Gangi, D., F. Lillo, and D. Pirino (2015). Assessing systemic risk due to fire sales spillover through maximum entropy network reconstruction. *Available at SSRN 2639178*.

Greenwood, R., A. Landier, and D. Thesmar (2015). Vulnerable banks. *J. Finan. Econ. 115*(3), 471–485.

Mastrandrea, R., T. Squartini, G. Fagiolo, and D. Garlaschelli (2014). Enhanced reconstruction of weighted networks from strengths and degrees. *New J. Phys. 16*(4), 043022.

Park, J. and M. E. J. Newman (2004, Dec). Statistical mechanics of networks. *Phys. Rev. E 70*, 066117.