Risk Attribution, Risk Budgeting and Portfolio's implied views: A factor approach

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Agenda

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1. Introduction
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3. Implied Views and Risk Attribution
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Introduction

The aim of the presentation is to propose a unique framework that can be used for risk attribution, risk budgeting and to monitor implied views on portfolios in order to have a complete control of coherence of the entire investment process of an Asset Manager.

Let us consider a simple factor model:

$$[1]\ r_t = Bf_t + \varepsilon_t$$

where: *r* is a Nx1 vector of returns on N assets

B is a matrix of coefficients N x K

f is a K x 1vector containing returns of factors

ε is a Nx 1 white noise vector.

Moreover, we suppose that:

$$E\left(f_t^{'}\varepsilon_t\right)=0 \ \text{ and } \varepsilon_t \approx \mathrm{N}\left(0,\Omega\right)$$

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In matrix terms, we can re-write equation [1] as follows:

$$[2]\ R = BF + E$$

From equation [2]:

$$[3]\ \sigma(R) = \sqrt{\mathrm{B}\Sigma\mathrm{B}' + \Omega}$$

Where Σ is the variance-covariance matrix of factors.

Consider now a portfolio vector (q) and a benchmark vector (b) which are linear combination of the N assets. We have a vector of bets on the N assets h=q-b and we can recover from equation [2] and [3], the following:

$$[4]\; h'R = h'BF + h'\mathrm{E}$$

And:

$$[5]\; \sigma\left(h'R\right) = \sqrt{h'\mathrm{B}\Sigma\mathrm{B}'h + h'\Omega h}$$

Where equation [5] identifies TEV.

Risk Attribution and Risk Budgeting in a factor model

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We can now decompose the active risk. Consider that we can decide if we want to view the risk using the asset's (N) dimension or the factor's (K) dimension or both (N x K), plus the idiosyncratic elements. We recover all the three kind of decomposition. First of all, we start from the asset dimension's risk decomposition.

$$[6] \quad \frac{\partial \sigma\left(h^{'} R\right)}{\partial h} = \frac{\left(B \Sigma B^{'} + \Omega\right) h}{\sigma\left(h^{'} R\right)}$$

For sake of simplicity we define σ=σ(h'R). Equation [6] contain the sensitivity of the portfolio active risk (TEV) w.r.t. changes in the vector of bets.

Risk Attribution and Risk Budgeting in a factor model

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We can decompose the sensitivity in two distinct elements: factor's sensitivity and specific sensitivity:

$$[7]\ \frac{\partial \sigma}{\partial h} = \frac{\left(B\Sigma B^{'}\right)h}{\sigma} + \frac{\Omega h}{\sigma}$$

If we pre-multiply equation [7] by a diagonal matrix <h>, we obtain the risk attribution at the asset level with the idiosincratic risk extrapolated from the total:

$$[8]\ \text{Risk Attribution} = \langle h \rangle \frac{\partial \sigma}{\partial h} = \langle h \rangle \frac{\left(B\Sigma B^{'}\right)h}{\sigma} + \langle h \rangle \frac{\Omega h}{\sigma}$$

Risk Attribution and Risk Budgeting in a factor model

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We can also calculate the TEV sensitivity due to change in factor exposures:

$$[9]\frac{\partial\sigma}{\partial B'h}=\frac{\partial h}{\partial B'h}\frac{\partial\sigma}{\partial h}\Rightarrow\frac{\partial\sigma}{\partial h}=\frac{\partial B'h}{\partial h}\frac{\partial\sigma}{\partial B'h}\Rightarrow$$

$$\Rightarrow B\frac{\partial\sigma}{\partial B'h}=\frac{B\Sigma B'h}{\sigma}+\frac{\Omega h}{\sigma}\Rightarrow$$

$$\Rightarrow\frac{\partial\sigma}{\partial B'h}=\frac{\Sigma B'h}{\sigma}+\left(B'B\right)^{-1}B'\frac{\Omega h}{\sigma}$$

Given that rank(B)=K .

We obtain the risk attribution due to factor exposures (see [10.a] for explanation of "z"):

$$[10]\langle B'h\rangle\frac{\partial\sigma}{\partial B'h}=\langle B'h\rangle\frac{\Sigma B'h}{\sigma}+z\langle B'h\rangle(B'B)^{-1}B'\frac{\Omega h}{\sigma}$$

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We must say that only if B is a full rank matrix and with N=K we have that the idiosincratic decomposition at the factors level sum up to the total Idiosincratic TEV, ie :

$$h' B(B' B)^{-1} B' \frac{\Omega h}{\sigma} = \frac{h' \Omega h}{\sigma}$$

In general N>>K, and thus we must correct the vector of factor "idiosincratic" sensitivities:

$$(B' B)^{-1} B' \frac{\Omega h}{\sigma}$$

by a scalar equal to:

$$[10.a] z = \frac{h' \Omega h}{\sigma} \Bigg/ h' B(B' B)^{-1} B' \frac{\Omega h}{\sigma}$$

Risk Attribution and Risk Budgeting in a factor model

We can also decompose the TEV using the two dimensions, factors and assets (having the idiosyncratic risk decomposition calculated from the second part of equation [8]):

$$[11.a]\text{TEV decomposition} = \left(\left(\left\langle B^{'} h \right\rangle \frac{\Sigma B^{'}}{\sigma} \left\langle h \right\rangle \right)^{'} \middle| \left\langle h \right\rangle \frac{\Omega h}{\sigma} \right)$$

The dimension of the matrix contained in equation [11.a] is N x (K+1), where the first N x K part contain the factor decomposition attributed to each single asset and the last column contains the idiosyncratic risk attributed to each single asset in the portfolio.

We can separate "pure" risk from the diversification effect in equation [11], using the following representation:

$$[11.b]\left(\left(\left\langle B' h\right\rangle \frac{diag(\Sigma)B'}{\sigma}\left\langle h\right\rangle\right)'\middle|\left(\left\langle B' h\right\rangle \frac{\Sigma B'}{\sigma}\left\langle h\right\rangle - \left\langle B' h\right\rangle \frac{diag(\Sigma)B'}{\sigma}\left\langle h\right\rangle\right)'\middle|\left\langle h\right\rangle \frac{\Omega h}{\sigma}\right)$$

The first N x K block represent the uncorrelated risk, the second N x K block the diversification effect and the last N x 1 vector contains the idiosyncratic risk.

Risk Attribution and Risk Budgeting in a factor model

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Once we have the two dimensional risk decomposition, we can recover easily a N x (K+1) matrix of sensitivities (**Marginal Contribution to Active Risk: MCAR**).

We know that the decomposition is obtained multiplying exposure times sensitivity and as we already know the result of this product from equation [11], we can recover the sensitivity matrix dividing each element of equation [11] by the corresponding element of the following matrix wich contain exposures:

$$\text{exposures} = \left(\left(B' \langle h \rangle \right)' \middle| h \right)$$

Risk Attribution and Risk Budgeting in a factor model

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Then:

$$[12.a]\mathrm{MCAR} = \left(\left(\left\langle B' h \right\rangle \frac{\Sigma B'}{\sigma} \left\langle h \right\rangle \right)' \middle| \left\langle h \right\rangle \frac{\Omega h}{\sigma} \right) ./ \left(\left(B' \left\langle h \right\rangle \right)' \middle| h \right)$$

Or, which is the same:

$$[12.b]\mathrm{MCAR} = \left(\left(u' \otimes \frac{\Sigma B' h}{\sigma} \right)' \middle| \frac{\Omega h}{\sigma} \right)$$

Where u is a unit vector of dimension N.

Implied Views and Risk Attribution

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For benchmark oriented products, consider the following utility function (when h'u=0, where "u" is a unit vector):

$$[13]\mathrm{U} = h^{'}Bf - \lambda\left(h^{'}B\Sigma B^{'}h + h^{'}\Omega h\right)$$

Where the utility is a positive function of the excess return of the portfolio vs benchmark and a negative function of the square of the TEV scaled by a positive risk adversion coefficient λ.

Let us suppose that our portfolio is a relative efficient portfolio vs benchmark with an expected IR equal to Θ. In case h'u≠0, we use the adjusted Info Ratio (we adjust the excess return by the borrowing/lending cost/return):

$$\theta = \frac{h'r - r_f h'u}{\sigma}$$

Implied Views and Risk Attribution

From [13] we can derive the First Order Condition (FOC) for a maximum:

$$[14]\frac{\partial U}{\partial h} = Bf - 2\lambda\left(B\Sigma B^{'}h + \Omega h\right) = 0 \Rightarrow r = Bf = 2\lambda\left(B\Sigma B^{'}h + \Omega h\right)$$

If we premultiply [14] by h' we obtain:

$$[15]h^{'}Bf = 2\lambda\left(h^{'}B\Sigma B^{'}h + h^{'}\Omega h\right) \Rightarrow 2\lambda = \frac{\theta}{\sigma}$$

Where σ is the TEV of the portfolio.

Implied Views and Risk Attribution

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Now we can recover an important result. We insert equation [15] in [14] and obtain a well known result:

$$[16] Bf = \theta\left(\frac{B\Sigma B' h}{\sigma} + \frac{\Omega h}{\sigma}\right)$$

Equation [16] is the vector of implied active alpha which can be obtained multiplying the expected Information Ratio for the Marginal Contribution to Active Risk.

If we derive the Utility Function w.r.t. the factor exposure (cfr. eq. 10.a), we obtain:

$$[17] f = \theta\left(\frac{\Sigma B' h}{\sigma} + z\left(B'B\right)^{-1} B' \frac{\Omega h}{\sigma}\right)$$

Implied Views and Risk Attribution

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At this point we have a direct link between risk attribution and implied views. Moreover, we can decide from which perspective we want to value the views (from an asset perspective or from the factor perspective) :

Factor's perspective implied active alpha (pure effect):

$$[17]\text{Factor's Implied Alpha} = \theta \frac{\Sigma B^{'} h}{\sigma}$$

Asset's perspective implied active alpha:

$$[18]\text{Asset's Implied Alpha} = \theta \frac{(B \Sigma B')h}{\sigma} + \theta \frac{\Omega h}{\sigma}$$

Implied Views and Risk Attribution

Where:

$$\theta \frac{\left(B\Sigma B^{'}\right)h}{\sigma}$$

Is the vector of implied alpha of the assets determined by their factor's exposure, while:

$$\theta \frac{\Omega h}{\sigma}$$

Is the implied alpha determined by the idiosyncratic component.

Implied Views and Risk Attribution

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Now we have all the elements to judge the coherence of the active portfolio with the tactical views of an Asset Management company.

Now suppose the company has views on factors: we can evaluate a portfolio using it's active exposure but also the factor's active implied alpha vector contained in equation [17].

The process will be in three steps:

1. Risk Budgeting (common factors, selection risk, currency risk)
2. Coherence of the portfolio positions with respect to which the Company gave an investment view (only if the active risk contribution is "significant")
3. Risk attribution of the sum of all the portfolio positions with respect to which the company has not given an investment view

Implied Views and Risk Attribution

When we sum risk contribution, we have the problem of how to calcultate the aggregate MCAR. We can calculate in three different way which imply three different meaning:

$$(\mathrm{i})MCAR = \frac{\sum_i |h_i| MCAR_i}{|h_i|} \quad (\mathrm{ii})MCAR = \frac{\sum_i h_i MCAR_i}{|h_i|} \quad (\mathrm{iii})MCAR = \frac{\sum_i h_i MCAR_i}{h_i}$$

We focus our attention on (i) and (iii).

While equation (i) is a sort of weighted average of implied returns, equation (iii) is the implied return of the strategy embedded in the disaggregated "area".

Implied Views and Risk Attribution

Even though equation (iii) maintains the properties that the risk=bet x MCAR, we prefer to use definition (i) because using (i) we can capture the implied view without mixing the information with the fact that the fund manager use it as an hedging (via h_i).

Moreover, it could be interesting to calculate the ratio of (iii) and (i) to obtain the "beta" of the embedded strategy vs the average return of the "sector".

Equation (i) reflects the BarraOne MCAR calculation methodology.

Implied Views and Risk Attribution

The following table shows how we can monitor an investment view by using the MCAR provided by BarraOne:

data source: BarraOne

Active Risk Contribution (bps)	Active Exposure (bet)	Marginal Contribution to Active Risk (MCAR)	Implied Investment View	Note
+	-	-	-	
+	-	+	+	
+	+	-	-	
+	+	+	+	
-	-	-	-	*hedging*
-	-	+	+	*hedging*
-	+	-	-	*hedging*
-	+	+	+	*hedging*

From the table we can see the correspondence between the sign of the MCAR and the implied view.

We need only to check the sign of the active risk contribution to be sure that the position is not an hedging.

An example

Let’s consider a balanced portfolio with benchmark 20% Cash, 45% Bond Govt, 10% Corporate Investment Grade, 25% Equity.

The Tracking Error Volatility from **BarraOne - BIM** is 1.00% and it is split between common factors (79 bps), selection risk (3 bps) and currency risk (18 bps).

Risk Source	Active Portfolio Risk Contribution	Active Portfolio Risk Contribution (in %)	Portfolio Risk Contribution	Benchmark Risk
Total Risk	1.00	100%	5.13	4.58
Local Market Risk	0.82	82%	4.83	4.54
Common Factor Risk	0.79	79%	4.82	4.53
Industry	0.54	54%	4.24	4.27
Style	0.01	1%	0.09	0.32
Term Structure	0.15	15%	0.30	1.88
Spread	0.07	7%	0.17	0.24
Emerging Market	0.02	2%	0.02	0.00
Selection Risk	0.03	3%	0.01	0.24
Currency Risk	0.18	18%	0.30	1.85

An example

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Let's consider the set of the investment views:

- ❑ underweight (negative view)
  - ✓ Cash
  - ✓ Bond Core (GER / US)
  - ✓ Bond Quasi Core (FRA / …)
  - ✓ Bond PIGS
  - ✓ Equity US
  - ✓ Equity Pacific ex-Japan
- ❑ overweight (positive view)
  - ✓ Bond Italy
  - ✓ Corporate High Yield
  - ✓ Equity Europe
  - ✓ Equity Japan
  - ✓ Equity Emerging Markets

		Market Portfolio	Active Portfolio	Portfolio
Cash	**Cash**	**10.0%**	**-6.1%**	**3.9%**
	Cash EUR	10.0%	-6.1%	3.9%
Bond Core	**Bond Core**	**17.0%**	**-1.7%**	**15.4%**
	Germany	10.3%	-0.6%	9.7%
	USA	6.8%	-1.1%	5.7%
Bond non Core	**Bond non Core**	**25.5%**	**-0.5%**	**25.0%**
	Quasi Core	7.7%	-2.0%	5.7%
	Italy	10.0%	6.5%	16.4%
	PIGS	7.8%	-5.0%	2.9%
Bond Inflation Linked	**Bond Inflation Linked**	**0.0%**		
	EMU Govt Inflation-Linked	0.0%	0.0%	
	US Treasuries Inflation-	0.0%	0.0%	
Corporate & Emerging Markets	**Corp & Em. Mkts**	**16.0%**	**2.7%**	**18.7%**
	Corporate IG	5.1%	0.0%	5.1%
	Corporate HY	7.0%	2.7%	9.7%
	Emerging Markets	4.0%	0.0%	4.0%
Equity	**Equity**	**31.4%**	**5.6%**	**37.0%**
	World Developed	26.5%	2.9%	29.4%
	Equity US	10.8%	-0.6%	10.2%
	Equity Europe	8.7%	4.5%	13.2%
	Equity Japan	4.2%	0.5%	4.6%
	Equity Pacific ex Japan	2.8%	-1.5%	1.4%
	Equity Emerging Markets	4.9%	2.7%	7.6%

An example

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Let's consider the *Bond Core* views. From the BIM factor exposure we can extract the **shift factor** (active) for Germany (-0.01) and US (+0.06).

Moreover, we have the active risk contribution of the two positions, which are +1 bp for Germany and -2 bps for US. The MCAR are negative: -41.91% and -35.31%.

Since the active risk contributions are marginal (less than 5% in percentage terms), we can not consider the application of investment views for these two positions.

In addition, we must also consider the contributions to active risk of **twist and butterfly factors** on these two countries.

		Investment Views			Exposure (source: BarraOne - BIM)			Contribution to Active Risk (source: BarraOne - BIM)	Contribution to Active Risk (in %)	Marginal Contribution to Active Risk
		Market Portfolio	Active Portfolio	Portfolio	PORTFOLIO	BENCHMARK	ACTIVE			
Bond Core	Bond Core	17.0%	-1.7%	15.4%	0.67	0.62	0.05	-0.02	-2%	
	Germany	10.3%	-0.6%	9.7%	0.46	0.47	-0.01	0.01	1%	-41.91%
	USA	6.8%	-1.1%	5.7%	0.21	0.15	0.06	-0.02	-2%	-35.31%
								0.01	1%	
	Twist Govt Germany				0.43	0.30	0.12	0.01	1%	7.10%
	Butterfly Govt Germany				0.11	0.05	0.06	0.00	0%	0.47%
	Twist Govt USA				0.08	0.06	0.03	0.00	0%	-0.63%
	Butterfly Govt USA				-0.06	-0.03	-0.03	0.00	0%	-1.33%

An example

Let's consider the *Bond non Core* views by aggregating the **shift factor** for Quasi Core (-0.23) and PIGS (-0.09) countries, while taking the one for Italy (-0.05).

The Quasi Core position contributes for 8 bps (8% of the active risk) and the PIGS position for 4 bps (4%), so we can evaluate the coherence of these active positions:

- Quasi Core → MCAR -35.99% is consistent w/underweight (negative) view
- PIGS → MCAR -45.46% is consistent w/underweight (negative) view

The Italy active position contributes for only 1 bp (1%), so we can not consider the application of investment views for that position.

	Investment Views				Exposure (source: BarraOne - BIM)			Contribution to Active Risk (source: BarraOne - BIM)	Contribution to Active Risk (in %)	Marginal Contribution to Active Risk
		Market Portfolio	Active Portfolio	Portfolio	PORTFOLIO	BENCHMARK	ACTIVE			
	Bond non Core	25.5%	-0.5%	25.0%	1.46	1.83	-0.37	0.13	13%	
Bond non Core	Quasi Core	7.7%	-2.0%	5.7%	0.65	0.89	-0.23	0.08	8%	-35.99%
	Italy	10.0%	6.5%	16.4%	0.57	0.61	-0.05	0.01	1%	-16.70%
	PIGS	7.8%	-5.0%	2.9%	0.24	0.33	-0.09	0.04	4%	-45.46%

An example

We have also to consider the **twist and butterfly factors** for the Quasi Core, Italy and PIGS countries.

We don't have any views on these factors, and the aggregate contribution to the active risks is marginal (-1 bp → -1%).

Note the negative active exposure on twist PIGS: 0.11 vs 0.13, and the positive MCAR 20.77%.

		Investment Views			Exposure (source: BarraOne - BIM)			Contribution to Active Risk (source: BarraOne - BIM)	Contribution to Active Risk (in %)	Marginal Contribution to Active Risk
		Market Portfolio	Active Portfolio	Portfolio	PORTFOLIO	BENCHMARK	ACTIVE			
								-0.01	-1%	
Bond non Core	Twist Govt Quasi Core				0.44	0.48	-0.04	0.00	0%	6.87%
	Butterfly Govt Quasi Core				0.00	-0.05	0.06	0.00	0%	-3.51%
	Twist Govt Italy				0.18	0.17	0.01	0.00	0%	9.53%
	Butterfly Govt Italy				-0.09	-0.05	-0.04	0.00	0%	-1.43%
	Twist Govt PIGS				0.11	0.13	-0.03	-0.01	-1%	20.77%
	Butterfly Govt PIGS				-0.01	-0.05	0.04	0.00	0%	-3.88%

An example

The active exposure on *Corporate High Yield* is +0.05 (**spread factors**) and it contributes 10 bps of active risk (10%):

- Corp HY→ MCAR +213.44% is consistent w/overweight (positive) view

					source: BarraOne - BIM			*source: BarraOne - BIM*		
	Investment Views				**Exposure**			**Contribution to Active Risk**	**Contribution to Active Risk (in %)**	**Marginal Contribution to Active Risk**
		Market Portfolio	**Active Portfolio**	**Portfolio**	**PORTFOLIO**	**BENCHMARK**	**ACTIVE**			
Corporate & Emerging Markets	**Corp & Em. Mkts**	**16.0%**	**2.7%**	**18.7%**	**1.10**	**0.98**	**0.11**	**0.09**	**9%**	
	Corporate IG	5.1%	**0.0%**	**5.1%**	0.91	0.92	-0.01	-0.02	-3%	191.02%
	Corporate HY	7.0%	**2.7%**	**9.7%**	0.05	0.01	0.05	0.10	10%	213.44%
	Emerging Markets	4.0%	**0.0%**	**4.0%**	0.14	0.06	0.08	0.00	0%	1.63%
	Spread Emerging Markets				0.05	0.00	0.05	0.02	2%	44.38%
	Twist Govt Emerging Markets				0.10	0.05	0.04	0.00	0%	-7.91%
	Butterfly Govt Emerging				0.03	0.03	0.01	0.00	0%	6.44%

An example

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The active exposure on *Equity US* and *Equity Europe* (**industry factors**) generate respectevely 16 bps and 21 bps, which are significant, so we can evaluate the coherence with respect to the investment views:

- Equity US → MCAR +17.61% is NOT consistent w/underweight (neg.) view
- Equity Europe → MCAR +16.78% is consistent w/overweight (pos.) view

Also the active position on *Equity Emerging Markets* is relevant (20 bps):

- Equity Emerging Mkts → MCAR +13.82% is consistent w/overweight view

	Investment Views				Exposure (*source: BarraOne - BIM*)			Contribution to Active Risk (*source: BarraOne - BIM*)	Contribution to Active Risk (in %)	Marginal Contribution to Active Risk
		Market Portfolio	Active Portfolio	Portfolio	PORTFOLIO	BENCHMARK	ACTIVE			
Equity	**Equity**	**31.4%**	**5.6%**	**37.0%**	**28.27**	**25.00**	**3.27**	**0.54**	**54%**	
	World Developed	26.5%	**2.9%**	**29.4%**	***24.74***	***22.92***	***1.83***	***0.34***	***34%***	***18.80%***
	Equity US	10.8%	-0.6%	10.2%	8.09	7.19	0.91	0.16	16%	17.61%
	Equity Europe	8.7%	4.5%	13.2%	14.72	13.44	1.28	0.21	21%	16.78%
	Equity Japan	4.2%	0.5%	4.6%	1.14	1.16	-0.02	0.00	0%	12.03%
	Equity Pacific ex Japan	2.8%	-1.5%	1.4%	0.79	1.12	-0.33	-0.03	-3%	8.26%
	Equity Emerging Markets	4.9%	**2.7%**	**7.6%**	***3.52***	***2.08***	***1.44***	**0.20**	**20%**	**13.82%**

An example

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The contribution to active risk of the **style factors** is 1 bp (1%), coming from a short exposure to size and a long exposure to volatility.

The contribution of the **selection risk** is 3 bps (3%).

Currency risk contributes for 18 bps (18%), mainly from short USD and JPY.

		Investment Views			Exposure (source: BarraOne - BIM)			Contribution to Active Risk (source: BarraOne - BIM)	Contribution to Active Risk (in %)	Marginal Contribution to Active Risk
		Market Portfolio	Active Portfolio	Portfolio	PORTFOLIO	BENCHMARK	ACTIVE			
Style	**Style Factors**							**0.01**	**1%**	
	Global Momentum				-0.01	-0.02	0.01	-0.01	-1%	
	Global Size				0.07	0.09	-0.02	-0.05	-5%	
	Global Value				0.00	0.02	-0.01	0.00	0%	
	Global Volatility				-0.03	-0.04	0.02	0.04	4%	
	Local Styles				0.07	0.09	-0.02	0.03	3%	
	Selection Risk							**0.03**	**3%**	
Currency	**Currency Risk**							**0.18**	**18%**	
	US Dollar				6.94	9.43	-2.49	0.10	10%	-3.99%
	Japanese Yen				3.39	4.07	-0.68	0.05	5%	-7.50%
	British Pound Sterling				5.42	5.15	0.26	0.00	0%	-0.44%
	Others				84.57	81.34	3.23	0.03	3%	

An example

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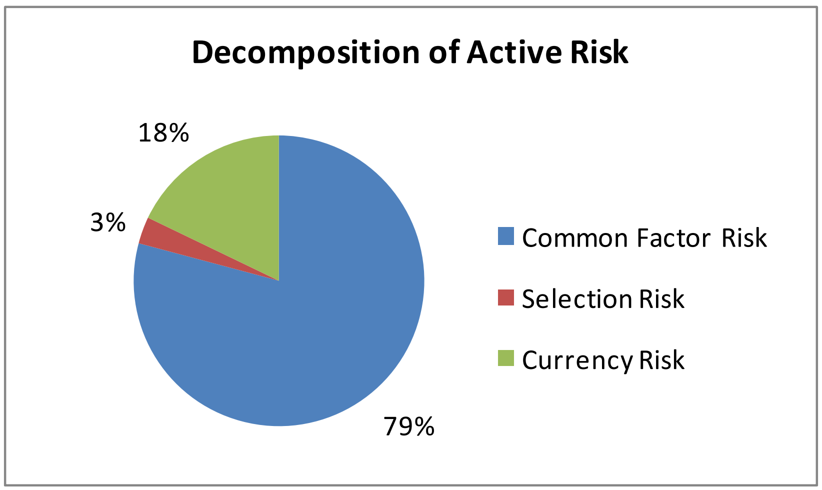
Coming back to our risk budgeting and risk attribution process:

Step 1 – Risk Budgeting

About 80% of the total active risk is allocated on the common factors; the currency component accounts for 18%, while only 3% is generated by security selection (or idiosincratic risk).



An example

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Step 2 – Coherence of investment views

The position on Equity US is NOT coherent with the investment view.

Asset Class	Investment View	Monitor
Bond Core (GER / US)	Negative	Not significant
Bond Quasi Core (FRA / …)	Negative	OK
Bond PIGS	Negative	OK
Bond Italy	Positive	Not significant
Corporate High Yield	Positive	OK
Equity US	Negative	NOT OK
Equity Europe	Positive	OK
Equity Japan	Positive	Not significant
Equity Pacific ex-Japan	Negative	Not significant
Equity Emerging Markes	Positive	OK

An example

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Step 3 – Risk contribution of the other positions

The active risk contribution of the portfolio positions with respect to which the company has not given an investment view is equal to 22%:

- ✓ Style factors → 1%
- ✓ Selection risk → 3%
- ✓ **Currency risk → 18%**

In particular the currency risk contribution is relevant and is generated mainly by the underweight of USD (-2.49% → 10 bps of active risk) and JPY (-0.68% → 5 bps of active risk).

Conclusion

This is just a proposal about how to use BarraOne – BIM risk analysis to monitor the investment process.

We need to do some more analysis in order to find the correct parametrization of the system (i.e., the level at which a risk contribution is considered "significant").

For an absolute return portfolio, we can use the same framework but we need to be careful because it is not a sufficient condition to have a positive weight on an asset class to have a positive implied view. In case negative correlations are involved sometimes we need a minimum weight to have a positive implied view. Let's start with the following general case were q is the vector of portfolio's weights:

$$[19]\sigma_P = \sqrt{q'\Sigma q}$$

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Given that all the element of q are positive, the condition to have an implied positive view on asset "j" is that it's correlation with the portfolio is greater then zero. If it is negative, the implied view is negative. Let's consider that the implied return, under the hypothesi of a Sharpe Ratio (in this cas we use a Sharpe ratio instead an info ratio) is:

$$[20] r = \frac{\Sigma q}{\sigma_P} = MCAR$$

We still call [20] Marginal Contribution to Active Risk as we compare the portfolio with a benchmark that is a cash account with no risk.

We can also easily recover the correlation of asset J with the portfolio in the following way:

$$[21]\rho_{j,P} = \frac{\{\Sigma q\}_{j,P}}{\sigma_j \sigma_P} \Rightarrow \rho_{j,P} = \frac{MCAR_j}{\sigma_j}$$

Then, we need to concentrate our attention to the covariance of the asset j w.r.t. the portfolio:

$$[22]Cov_{j,P} = q_j \sigma_j^2 + \sum_{i \neq j} q_i \sigma_{i,j}$$

Appendix

We search the value of q for which the correlation is equal to zero and this implies that also the implied return is zero. This means that below this threshold the asset is an hedging while above is a positive bet. We can recover two different kind of thresholds:

(1) Marginal Hedging Threshold (MHT);

(2) Total Hedging Threshold (THT).

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(1) Marginal Hedging Threshold (MHT): we investigate the value of q for which the correlation is zero, leaving unchanged all the other portfolio's weights.

$$[22] Cov_{j,P} = q_j \sigma_j^2 + \sum_{i \neq j} q_i \sigma_{i,j} = 0 \Rightarrow PHT = -\frac{\sum_{i \neq j} q_i \sigma_{i,j}}{\sigma_j^2} \Rightarrow$$

$$\Rightarrow MHT = -\frac{MCAR_j \sigma_P - q_j \sigma_j^2}{\sigma_j^2}$$

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(2) Total Hedging Threshold (THT): we investigate the value of q for which the correlation is zero, financing the increasing/decreasing position on q by an equivalent decreasing/increasing in weights in all the other portfolio's asset:

$$[23] THT \Rightarrow (h_j + \Delta)\sigma_j^2 + \sum_{i \neq j}\left(h_i - \Delta \frac{h_i}{1-h_j}\right)\sigma_{i,j} = 0 \Rightarrow$$

$$\Rightarrow THT = h_j + \Delta \text{ where } \Delta = -\frac{\mathrm{MCAR}_j \sigma_P}{\sigma_j^2 - \frac{1}{1-h_j}\left(MCAR_j \sigma_P - \sigma_j^2 h_j\right)}$$

In addition to MHT and THT, we can also calculate a measure of Maximum Hedging Weight (MHW). This quantity measures the weight of a bet or an absolute position that maximise the hedging contribution of the asset classes. In formulas, we search an h_i such that minimize equation [24]:

$$[24]\min_{h_i} h_i \left\{ \frac{\Sigma h}{\sigma} \right\}_i$$

If we derive [24] for h_i we obtain the following:

$$[25] MHW = -\frac{\sum_{j \neq i} h_j \sigma_{ij}}{2\sigma_i^2} = \frac{\left\{ \frac{\Sigma h}{\sigma} \right\}_i \sigma - h_i \sigma_i^2}{2\sigma_i^2}$$