Innovation, specialization and growth in a model of structural change

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1 Motivation

1.1 Technical progress, size of the market, and labor specialization

The progress in technology and science applied to industry has brought about an extraordinary expansion of implements assisting labour in producing final commodities.

The growth of the size of the market, in a Smithian sense, lies at the heart of this movement but evidence also suggests that there is a narrow relationship between increasing productivity and the pattern of final demand.

1.2 High and low priority goods

Low productivity and low income economies are by necessity constrained to afford only staple consumption goods required to support a basic livelihood

As productivity rises and income per head is augmented, the economy is reshaped to accommodate a demand pattern made up of goods of lesser priority, possibly allowing greater comfort and affluence

The link between demand patterns and income levels has been stylized by the well-known Engel's curves showing that as income rises the weight of some commodities increases whilst that of others declines as demand is driven to saturation for the latter and as acceleration for the former occurs

1.3 The drive to innovate

The aim of this paper is to investigate the nexus between demand patterns, hallmarks of development stages, and innovation as it stems from research efforts and the extent of specialization.

The model we consider strives to put the latter at the centrestage of our analysis to determine the productivity growth allowing for income per head increases generating demand shifts and hence structural change.

The main source of growth is innovation the burden of which is mainly laid upon intermediate (capital) goods manufacturers.

It is innovation that leads to productivity growth and thus on growth of real income. Innovations, however, are idiosyncratic events that require investment in specific resources.

2 Consumption pattern

Consider an economy with

- J differentiated goods
- with prices $p_{1,t}, \ldots, p_{J,t}$ at time t
- and populated by $\mathcal{L}$ consumers.
  The individuals' intertemporal discount factor coincides with the constant interest rate r

2.1 The consumer's problem

The consumer's problem is a problem where, for given prices $p_{1,t}, ..., p_{j,t}, ..., p_{J,t}$ and income R_i, $i = 1, 2...\mathcal{L}$, an optimal decision has to be made concerning

- how many goods are to be consumed
- and how much of each is to be consumed

a variety and quantity choice problem.

2.2 The individual demand function

Consuming $y_{i,h,t}$ of good h at time t for $h = 1, ..., j$, yields to the individual i an instantaneous utility

$$u_i(j,t) = Log\left(C_{i,j}\right) + \alpha_j \sum_{h=1}^{j} Log\left(y_{i,h,t}\right)$$

where $C_{i,j}$ is a weight that accounts for individual i's impatience to consume good j

2.3 Quantity and Variety Choice

For given prices $p_{1,t}, ..., p_{J,t}$, and income R_i, the individual i's problem can be written as a sequence of **quantity problems**

$$U_i(j,t) = \max_{y_{i,1,t} \cdots, y_{i,j,t}} u_i(j,t) \quad \text{s.t.} \sum_{h=1}^{j} p_{h,t} y_{i,h,t} \leq R_i \tag{1}$$

and a **variety problem**

$$max_{j \in \{1,...,J\}} U_i(j,t) \tag{2}$$

2.4 Sufficient Conditions and Demand Function

- The difference $\alpha_{j+1} - \alpha_j$ is positive and non-decreasing in j ;

- constants $C_{i,j}$ for $i = 1, ..., \mathcal{L}$ and $j = 1, ..., J$ satisfy the condition that $\chi_i(j) \equiv \frac{C_{i,j+1}}{C_{i,j}} \frac{\left(\frac{R_i}{j+1}\right)^{(j+1)\alpha_{j+1}}}{\left(\frac{R_i}{j}\right)^{j\alpha_j}}$ is non-increasing in j.

- prices are such that $p_{j+1,t} > p_{j,t} > 1$ for each $t \geq 0$.

2.4.1 Demand function

Let $j^*_{i,t}$ be the optimal variety at time t, that is $U_i\left(j^*_{i,t}, t\right) > U_i\left(j, t\right)$ for each j other than $j^*_{i,t}$, then

$$y_{i,j,t} = \begin{cases} \frac{1}{j^*_{i,t}} \frac{R_i}{p_{j,t}} & \text{for each } j \leq j^*_{i,t} \\ 0 & \text{for each } j > j^*_{i,t} \end{cases} \quad (3)$$

3 The Production Structure

We discuss an economy in which there are J industries. They are composite entities of technologically vertically integrated production.

These J industries are each composed by a **final sector** populated by a large number of firms, operating in a freely competitive environment such that the prevailing prices drive their profits to zero.

By contrast, in each industry j, k_j sectors engage in producing an industry **specific intermediate good** which embodies the latest technology and are the carriers of innovation.$(j = 1, 2....J)$

Each sector features an **incumbent** monopolist earning a positive profit and a searching-to-innovate **follower**.

3.1 The final sectors

The J industries final goods producing sectors avail themselves of a simple supply structure.

The following is the linear production function featuring inputs that are all strictly complementary and that all firms in any industry j apply to produce their specific output:

$$y^s_{j,t} = \min \left\{ a^1_{j,t} x^1_{j,t}, a^2_{j,t} x^2_{j,t} \ldots a^{k_{j,t}}_{j,t} x^{k_{j,t}}_{j,t}, b_{j,t} l^y_{j,t} \right\} \quad ; \tag{1}$$

It is important to note that $k_{j,t}$ sets the number of specialized sectors that at time t are required to produce intermediate inputs.

3.2 Intermediate goods producing sectors

The supply structure of these sectors is simpler.

We normalize the production process of intermediate goods in such a manner that they all require the same amount of labour for one unit to be produced.

Thus, the following production function holds, $\forall k, j$:

$$x_{j,t}^{k} = \eta l_{j,t}^{k} \quad , \tag{2}$$

$l_{j,t}^{k}$ being the amount of manpower employed for the purpose of producing $x_{j,t}^{k}$ units of the intermediate good k in industry j at time t.

3.2.1 Monopoly power and prices of the intermediate goods

Monopolists, owners of up-to-date technologies, enjoy a positive profit that equals:

$$\pi^k_{j,t} = u^k_{j,t} x^k_{j,t} - w l^k_{j,t} \quad . \tag{3}$$

Assumption 1 The price of an intermediate good k , $u^k_{j,t}$, is set according to a mark-up $c^k_{j,t}$.

$$u^k_{j,t} = (1 + c^k_{j,t}) \frac{1}{\eta} w \quad . \tag{4}$$

that we later determine.

4 Innovations in the Intermediate Good Sectors

This simple production structure is stressed by the occurrence of innovations. The latter are introduced by old and new producers of intermediates and increase final sectors productivity. To simplify, it is assumed that when an innovation occurs, its effects on productivity spread equally on both intermediate and labour requirements.

4.1 The protagonists of innovation

a) Followers of monopolists-in-charge who, by R.&D., attempt to improve existing intermediates and gain a dominant position.

b) Agents of specialization, normally situated in intermediate producing sectors who, by spin-offs, introduce new intermediates to the existing chain.

c) Agents of rationalization who, by eschewing some intermediates, shorten the extant chain and cut costs.

4.2 Vertical innovations

A first kind occurs as a consequence of efforts made by erstwhile monopolists who were ousted by current incumbents.

Assumption 2 An innovation occurring at time t in sector k of any industry j raises the productivity of all inputs $b_{j,t}$ and $a^k_{j,t}$, $\forall k$ by a factor λ.

$$\begin{aligned} a^k_{j,t} &= a^k_{j,t-}e^{\lambda} \quad , k = 1, 2, .., k_{j,t}; \\ b_{j,t} &= b_{j,t-}e^{\lambda} \quad . \end{aligned} \tag{5}$$

The current inputs $a^k_{j,t}$ and $b_{j,t}$ are recorded to be the result of past innovations at each time step increasing productivity by a constant λ.

4.2.1 Productivity index

The average ratio of indirect to direct labour requirement that is specific to each industry j

$$\delta_{j,t} = \frac{1}{k_{j,t}} \sum_{k=1}^{k_{j,t}} \frac{b_{j,0}}{\eta a_{j,0}^{k}} \quad . \tag{6}$$

plays an important role in the following analysis.

4.3 Innovations by specialization and rationalization

The innovation process, however, is not confined to the enhancement of existing inputs productivity, but has evolved by deepening the utilization of means of production entering final goods, the process on which it has been grounded has been hallmarked by intertwined events of both specialization and restructuring.

Events of this kind can be accounted for, in the former case, by the lengthening in some industries of the $k_{j,t}$ strings of intermediate inputs and, in the latter one, by shortening in some others..

4.3.1 Specialization

Assumption 3 (i) Let t denote the time of specialization, then:

a) at each step, the deepening of specialization brings about an overall productivity increase equal to λ.

$$\begin{aligned} k_{j,t} &= k_{j,t-} + 1 \\ a^k_{j,t} &= a_{j,t-} e^{\lambda} \ , \ k = 1, 2, .., k_{j,t} - 1 \\ b_{j,t} &= b_{j,t-} e^{\lambda} \ \ ; \end{aligned} \tag{7}$$

b) the average ratio of indirect to direct labour (6) requirement does not increase as a more specialized technology is introduced $\delta_{j,t} \leq \delta_{j,t-}$;

4.3.2 Rationalization

Assumption 3 (ii) Let t denote the time of rationalization, then:

a) the process of rationalization is merely cost-cutting: when it occurs the number $k_{j,t}$ is reduced;

c) as soon as either a specialization or a rationalization occurs, followers are able to adopt it and adapt their outdated technology thus keeping their gap from widening.

5 Prices, Profits and the Mark-up

Assumption 4. The absence of arbitrage opportunities among different sectors of the same industry insures that the employment of the same quantity of labour affords an equal profit flow

$$\pi_{j,t}^{k} = \pi_{j,t}^{k'} \quad k, k' = 1, ..., k_{j,t} \ , \ j = 1, .., J \quad .$$

Assumption 4 allows us to distinguish two different mark-up components: an industry wide term $c_{j,t}$ and a sector specific correcting factor $a_{j,0}^{k}$ which is linked to the k-good productivity.

5.0.3 Mark-up

The size of the sector specific component is to be such that the productivity increase be entirely appropriated by the entrant and the old incumbent's profits be driven to zero

Lemma 1 *The mark-up of intermediate good producer k of industry j is*

$$c_{j,t}^{k} = c_{j,t} a_{j,0}^{k} \quad , \tag{8}$$

where the industry wide mark-up is

$$c_{j,t} = \frac{\eta}{b_{j,0} k_{j,t}} \left(1 + k_{j,t} \delta_{j,t}\right) \left(e^{\lambda} - 1\right) \quad . \tag{9}$$

5.1 Prices and profits

Remark 1 *On account of the non-arbitrage Assumption 5, the profit of an intermediate good producer in any industry j depends only on the specific industry, but not on the specific intermediate good k*

$$\pi_{j,t} = \pi^k_{j,t} = \frac{1}{k_{j,t}} \left(1 + \delta_{j,t} k_{j,t}\right) \left(e^{\lambda} - 1\right) w l^y_{j,t} \quad , \tag{10}$$

while the price of a final good j becomes simply

$$p_{j,t} = \frac{w}{b_{j,t}} \left(1 + \delta_{j,t} k_{j,t}\right) e^{\lambda} \quad . \tag{11}$$

5.2 Innovations and spin-offs

The appearance of new sectors carries with it the burden of a new production process encumbering the economy with more employment, a new technique and yet another monopolist enjoying exclusive ownership rights upon it.

Some conditions must be satisfied for the lengthening of the process to be feasible and further specialization take place. A productivity increase, say at time t, must translate into lower prices

$$p_{j,t} \leq p_{j,t-} \tag{12}$$

Condition (12) is met provided that the increase in productivity λ is sufficiently high as to more than offset the increase in real direct and indirect labour costs implied by the lengthening of the specialization string.

6 Effective Demand, Profits and Employment

Our assumptions on the demand yields that each industry j collects a different share β_j of total aggregate output (1) (2) (3)

$$\beta_{j,t} = \sum_{i=1}^{\mathcal{L}} I_{j \leq j^*_{i,t}} \frac{R_i}{j^*_{i,t} Y_t}$$

where $I_{j \leq j^*_{i,t}}$ is an indicator function, indicating $\mathbf{1}$ if $j \leq j^*_{i,t}$.

In equilibrium of aggregate demand Y_t. At any point of time t, real demand and supply equilibrium simply is:

$$y^s_{j,t} = y^d_{j,t} = \beta_{j,t} \frac{Y_t}{p_{j,t}} \quad . \tag{16}$$

6.1 Effective demand, productivity, and profits

The k-th sector realized profits (10) in this industry are, in consequence, a mere proportion of aggregate output:

$$\pi_{j,t} = \left(1 - e^{-\lambda}\right) \frac{1}{k_{j,t}} \beta_{j,t} Y_t \quad , \tag{14}$$

showing that the sector-wise flow of profits, given the productivity rate of increase and the demand share, depends only on the size of aggregate output and the extent of specialization.

6.2 Employment

Assumption 5. The size of overall employment $\mathcal{L}$ is kept constant

$$\mathcal{L} = L_t^y + L_t^x + H_t \quad .$$

Here

L_t^y is the total **final-good employment**

L_t^x is the total **employment of intermediate goods producing sectors** across all industries

$H_t = \sum_{j=1}^{J} h_{j,t}^k$ is the employment of manpower that followers use to conjure up the next round of innovation,

where $h_{j,t}^k$ measures the **innovative effort** of firm k in industry j at time t.

7 Innovation, Growth and Structural Change.

In this section we specify the innovation processes and determine the economy's stationary state as well as its expected growth rate. **Three different innovation events** are actually dealt with, two of them being concomitant.

The first is a **vertical innovation,** a firm-specific occurrence, that results from followers' researching efforts

The second and third are a **specialization** spin-off and a **rationalization** process occurring in consequence of a demand shift that is assumed to happen according to an arrival rate μ_d: the former where the demand share becomes larger, the latter where it contracts.

7.1 Arrival rate of a vertical innovation

A vertical innovation comes to pass with a Poisson arrival rate.

Since this rate is increasing with the number of employees (h) hired to carry out this process, we normalize it as h such that the probability of an innovation event in a period dt of time is hdt.

These efforts in the same time period imply a cost that for simplicity's sake is rendered by $C(h) = \frac{a}{2}h^2 + \frac{F}{2}$.

7.2 Value of an innovation

If equilibrium prevails, innovators in industry j will reckon that their expected flow of profits based on the likely value of their innovation is, given a discount rate r,

$$rV_{j,t} = \pi_{j,t}w - h_{f,t}V_{j,t} - \mu_d V_{j,t}\frac{\max\left\{\Delta k_{j,t}, 0\right\}}{k_{j,t}} \tag{15}$$

where $\Delta k_{j,t} = k_{j,t} - k'_{j,t}$ is the difference between the number $k_{j,t}$ of intermediate producers in industry j at time t and their expected number $k'_{j,t}$ after the demand shock.

Note that $h_{f,t}$ denotes the vertical innovation arrival rate as it applies to followers-to-be.

7.2.1 Optimal research effort

Maximization of the net expected value of a vertical innovation for a follower in the intermediate sector of industry j reads

$$W_{j,t} = \max_{h} \; hV_{j,t} - \frac{a}{2}h^2 - \frac{F}{2} \quad \text{for } j = 1..., J \; ; \tag{16}$$

here we assume that firms do not internalize the effect of their own research efforts on the arrival rate μ_d.

The J first order conditions resulting from the maximization problems (16) yield the followers' innovative efforts $h_{j,t}$ as functions of the number of monopolists in each industry and of the expected efforts of future and present competitors.

7.3 Stationary state

We first define the stationary state.

Definition 1 *A stationary state is the state in which the demand pattern does not and is not expected to change over time.*

7.3.1 Characterization of the economy's stationary state.

Proposition 2 *In the stationary state the number of intermediate goods sectors in industry j amounts to:*

$$k_j = \beta_j \left(1 - e^{-\lambda}\right) \frac{Y}{F} \quad , \tag{17}$$

from which the overall total number of sectors:

$$\bar{k} = \sum_{j=1}^{J} k_j = \left(1 - e^{-\lambda}\right) \frac{Y}{F} \tag{18}$$

and aggregate output is

$$Y = \frac{\mathcal{L}}{\frac{1}{we^{\lambda}} + \sqrt{\frac{1}{Fa}} \left(1 - e^{-\lambda}\right)} \quad . \tag{19}$$

7.4 The stationary state growth rate

The economy growth is essentially due to productivity growth which is, in turn, explained by the innovations that followers conjure up in their strive to oust reigning monopolists, a feat that is achieved thanks to investment in research and development.

Proposition 3 *The stationary state growth rate is*

$$g_{Y_R} = \sqrt{\frac{F}{a}} \sum_{j=1}^{J} \beta_j \left(e^{\lambda \beta_j f(\mathcal{L})} - 1 \right) \quad . \tag{20}$$

7.4.1 Remark 1

The growth rate **depends on total employment** $\mathcal{L}$, effectively a proxy of the extent of the market.

In an economy in which it is held constant, the sum of the intermediate sectors, $\overline{k}$, also remains constant, specialization having gone further where demand shares had increased and restructuring eased sectorial costs where they had declined.

A larger $\mathcal{L}$ implies a greater number of intermediate sectors hence a higher aggregate growth rate: a larger extent of the market in a Smithian sense deepens overall specialization and enhances the economy's long-run growth rate.

7.4.2 Remark 2

The growth rate **depends on the distribution of the intermediate sectors** amongst the various industries.

As the development process unfolds assigning greater weights to goods that are less essential with real income growth, the impact of innovation owing to monopolists' followers' research and development efforts is spread more evenly over a larger number of industries. The implication is that the overall growth factor is lessened on account of a more balanced distribution of $\overline{k}$ over the entire number of industries.

If the development process gets under way, the growth rate of the less developed economies rises whilst that of the more developed ones slows down generating a process of convergence.

7.4.3 Remark 3

An economy that manages to concentrate its aggregate output on fewer industries, other things being equal, achieves higher aggregate growth for the simple reason that specialization is also more concentrated: the same $\bar{k}$ distributed on fewer $j's$.

This configuration may, for instance, occur in economies that, in spite of possessing a high real income per capita, through foreign trade have specialized in the production of a relatively small number of goods that it exports while importing many more allowing comparative advantage and higher growth.

7.5 Traverse dynamics

7.5.1 A simplified case

We assume the existence of only two industries, industry 1 initially producing luxury goods and industry 2 producing basic ones, and by normalizing the initial magnitudes of their sector productivities as to be equal at $t = 0$. Hence

$$a^k_{j,0} = a_0 \text{ for } k = 1, 2, \ldots \text{ and } j = 1, 2 \tag{21}$$

and

$$b_{j,0} = b_0 \text{ for } j = 1, 2 \tag{22}$$

Given this assumption, the index (6) δ_j no longer depends on j and $\delta = \frac{1}{\eta}\frac{b_0}{a_0}$.

7.5.2 Unfolding of the traverse path

We now proceed to study a system in which several demand shocks, say T, sequentially occur. In consequence the economy evolves as a sequence $\tau = 0, 1, ..., T$ of successive meta-stationary state equilibria , each beginning and ending with a demand shock.

Definition 2 *A meta-stationary state equilibrium is a state in which the demand pattern does not change but in which it is expected to change over time.*

7.5.3 Arrival rate of a demand shock

Since demand shocks are more frequent the more rapid the decline of the basic good price $p_{2,\tau}$ is and eventually the more frequent vertical innovations are, a suitable formalization is the following

Assumption 6. The arrival rate of demand shocks μ_d directly depends on the (expected) average research efforts $\overline{h}_2$ of firms producing the high priority good $\mu_d(h) = \overline{\mu}_d \overline{h}_2$.

7.6 Characteristics of the traverse path

Assuming that at each stage agents are able to exactly forecast the number of monopolists in the subsequent stage, we can prove the following results, in the limit for $\delta \to 0$.

Proposition 4 *(a) The sequence* $\left\{k_{2,\tau}\right\}_{\tau=1}^{T}$ *is strictly decreasing.*

(b) The sequence $\left\{k_{1,\tau}\right\}_{\tau=1}^{T}$ *is strictly increasing.*

(c) For every $\tau = 1, ..., T-1$, $L_\tau > L_T$.

7.6.1 Remark

It is important to note that this proposition implies that the level of manpower employed in supporting the innovation process declines during the traverse.

This effect is due to the uncertainty that is generated by demand shifts and thus by the likelihood that rationalization may wipe out some intermediate producing sectors.