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Understanding Risks
In Structured Products

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December 2011, Scuola Normale Superiore - Pisa

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1. Common Denominator in SP Business

- **Structured product business pillars:**
  - **Marketing/structuring;**
  - **Trading/risk managing;**
  - **Quantitative modeling;**
- **Marketing/structuring: provide products to meet investors' financial needs & risk appetites;**
- **Trading/risk managing: hedge & mitigate risks;**
- **Quantitative modeling: develop quantitative models to price the cost of hedging risks;**

Risk is the Common Denominator

	Capital	Return	Risks
Growth Products	Secured	Linked to Markets	Option Risks (Impact Return)
Income Products	Not Secured	Much Enhanced Yield/Income	Option Risks (Impact Capital)
CPPI (Fund)	Protected But Not Secured	Linked to Markets	Gap Risks (Impact Both)

Quantitative Models

- **Quantitative models: mathematical tools to price the Risks and their hedging costs;**
- **Good quantitative models:**
  - **Able to calibrate to the markets properly;**
  - **Able to capture key risk exposures;**
  - **Numerically stable for pricing;**
  - **Numerically stable for generating Greeks;**
  - **Computationally efficient.**

2. Key Risk Characteristics of Structured Products

- **Partial Differential Equation (PDE):**

$$\frac{\partial V}{\partial t} + (r - q) \cdot S \cdot \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 \cdot S^2 \cdot \frac{\partial^2 V}{\partial S^2} = r \cdot V$$

- **Risks Embedded in Options:**
  - **Time decay – Theta;**
  - **Spot – Delta;**
  - **Spot – Gamma;**
  - **Volatility – Vega;**
  - **Rate – Rho.**

“Exotic” Risks

- **“Exotic” Risks Embedded in Options:**
  - **Dividend Risk;**
  - **Quanto Risk;**
  - **Volatility Skew/Smile Risk;**
  - **Gap Risk;**
  - **Hybrid Risk;**
  - **Correlation Risk.**

Risks That Supposed to be Trivial

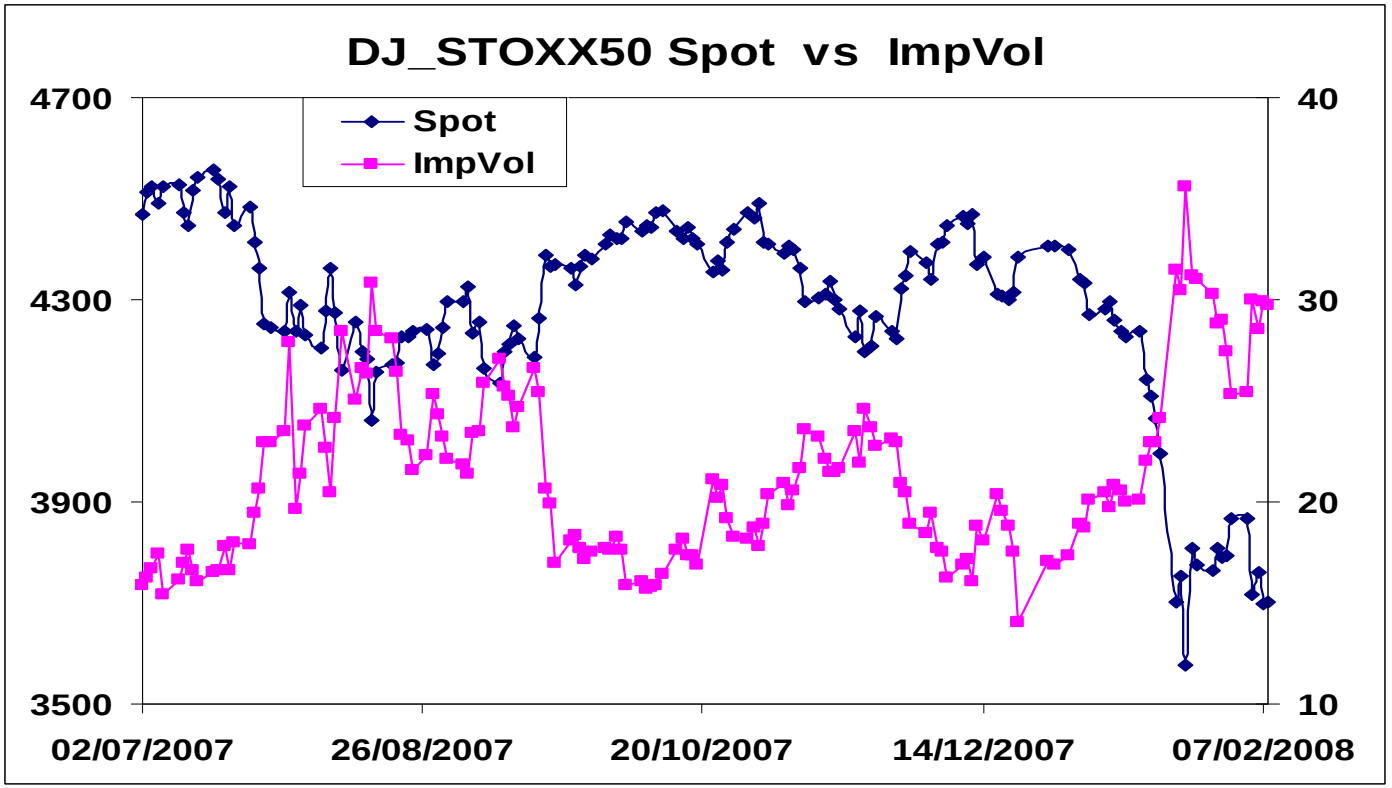
- **Dividend Risk:**
  - **Most banks use deterministic sets of forecast dividends in pricing models;**
  - **During 2008 crises, banks had to mark down the dividends, resulted in substantial losses;**
- **Quanto Risk:**
  - **Most banks use the simple quanto adjustment;**
  - **Again, during 2008 crises, banks' equity positions lost lots of money on quanto (FX) exposures.**

Quanto Hedge Becomes Difficult …

- **Both equity and FX vol shoot up;**
- **Correlation shoots up;**
- **Stochastic vol effect becomes significant;**
- **FX smile comes into play.**

Risks that supposed to be insignificant became Massive!

Volatility Skew/Smile Risk



- **When market crashes, volatility shoots up!**

Delta in the Presence of Skew

- **Equity skew effectively illustrates the correlation between Spot and Volatility;**
- **Option delta is a function of skew:**

$$\Delta = \frac{\partial V}{\partial S} + \frac{\partial V}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial S}$$

- **Skew dynamics:**
  - **Sticky delta (ATM vol moves with spot);**
  - **Sticky strike (Black-Scholes delta);**
  - **Sticky local vol (Static local vol, e.g. in PDE);**

Delta Hedge Under Skew Dynamics

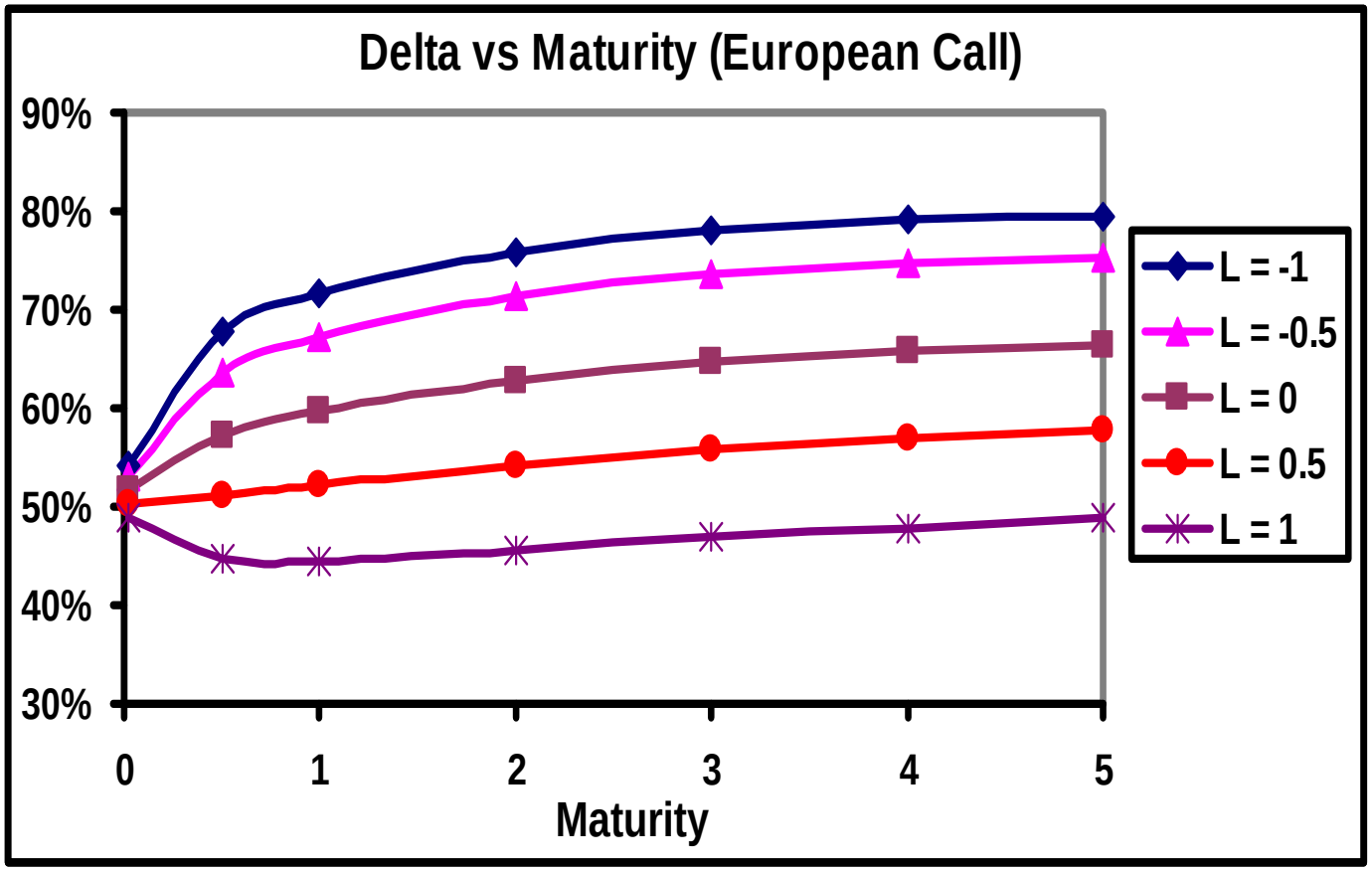
- **Re-writing option delta:**

$$\Delta = \frac{\partial V}{\partial S} + \frac{\partial V}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial S}$$

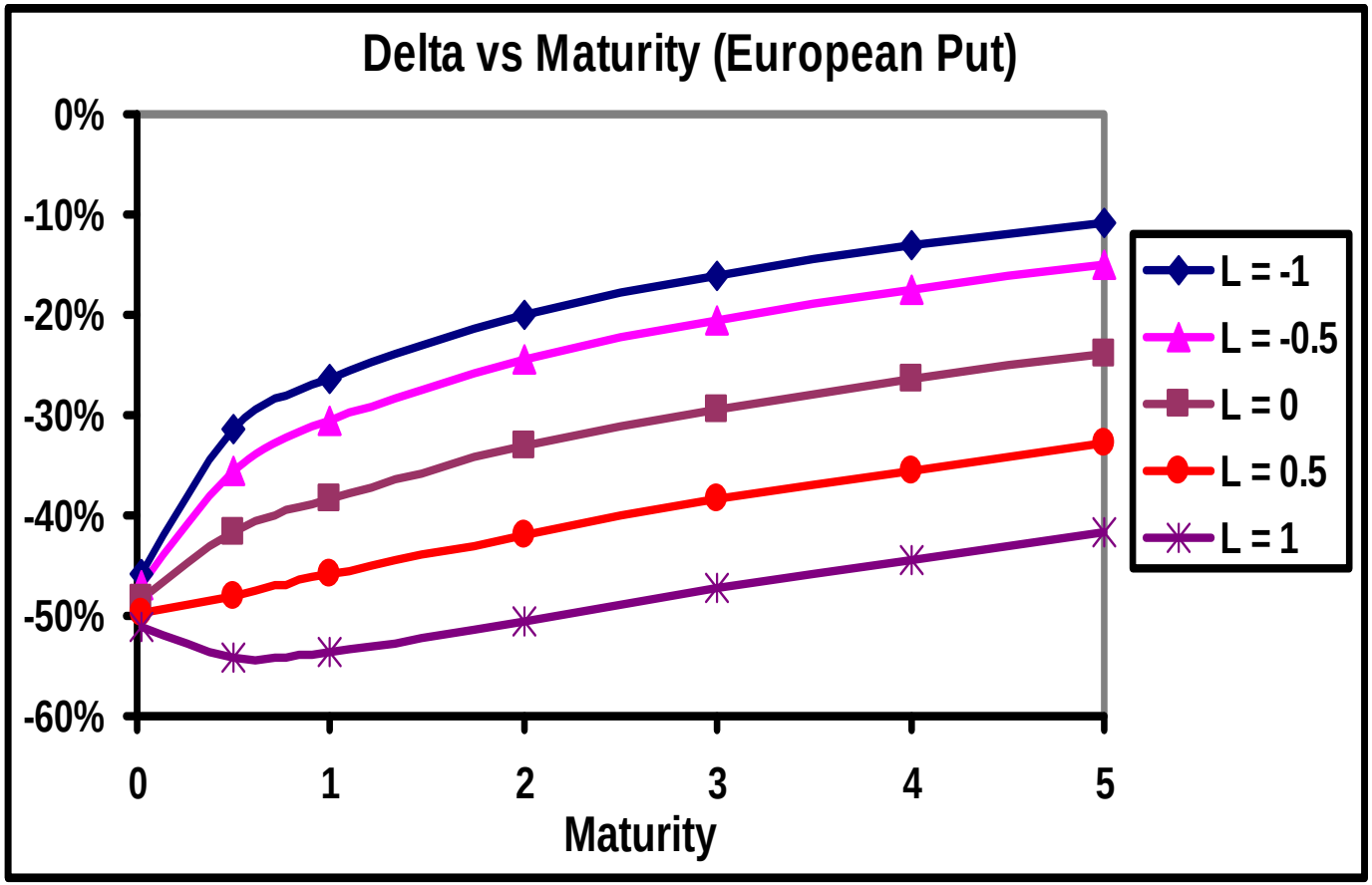
$$= \Delta_{BS} + vega_{BS} \cdot \left.\frac{\partial \sigma}{\partial S}\right|_{Surface} \cdot L(t)$$

- **Delta under skew dynamics:**
  - ***L(t) = -1*, sticky delta, real delta > BS delta;**
  - ***L(t) = 0*, sticky strike, real delta = BS delta;**
  - ***L(t) = 1*, sticky local vol, real delta < BS delta;**

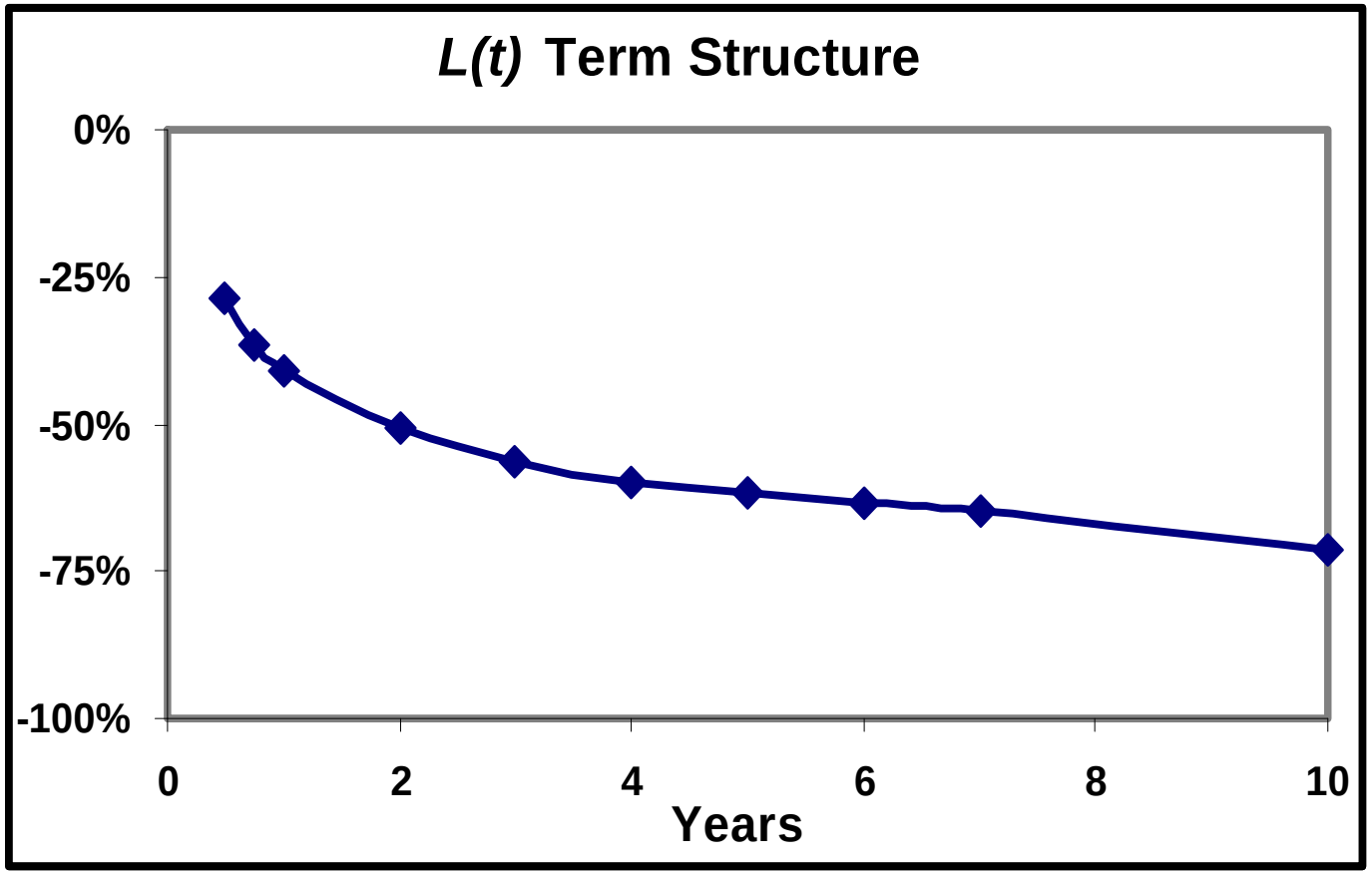
Delta Under Skew Dynamics (CALL)



Delta Under Skew Dynamics (PUT)



An Example of *L(t)* Term Structure



Skew Risks in Illiquid Markets

- **Skew risks are real:**
  - **Skew represents spot-vol correlation;**
  - **When market moves, volatility moves too!**
- **In illiquid markets (e.g. emerging markets):**
  - **Option quotes are scarce, even for ATM;**
  - **Very difficult to obtain implied vols at various strikes;**
  - **Very little market information on volatility skew;**
  - **But the skew risks STILL exist and are REAL!**

Gap Risks in CPPI

- **CPPI uses *GC* measure to dynamically re-balance risky assets (e.g. shares) and risk free assets (e.g. cash);**
- **Gap Condition (*GC*):**

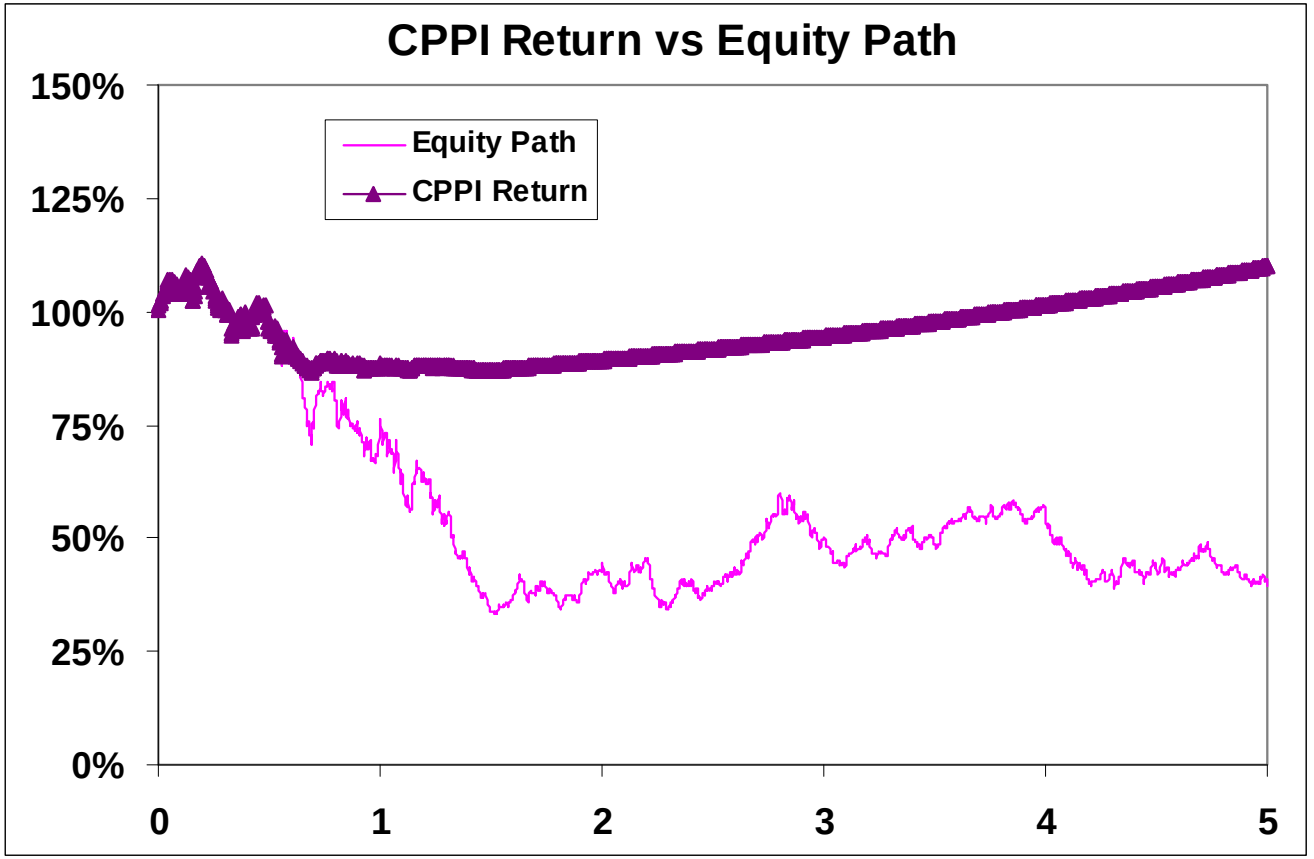
$$GC = V_t - G_T \cdot DF_t^T$$

- **Gap Condition is simply the gap between the current fund value and promised guarantee (pv_ed);**
- **It is effectively an investment safety buffer.**

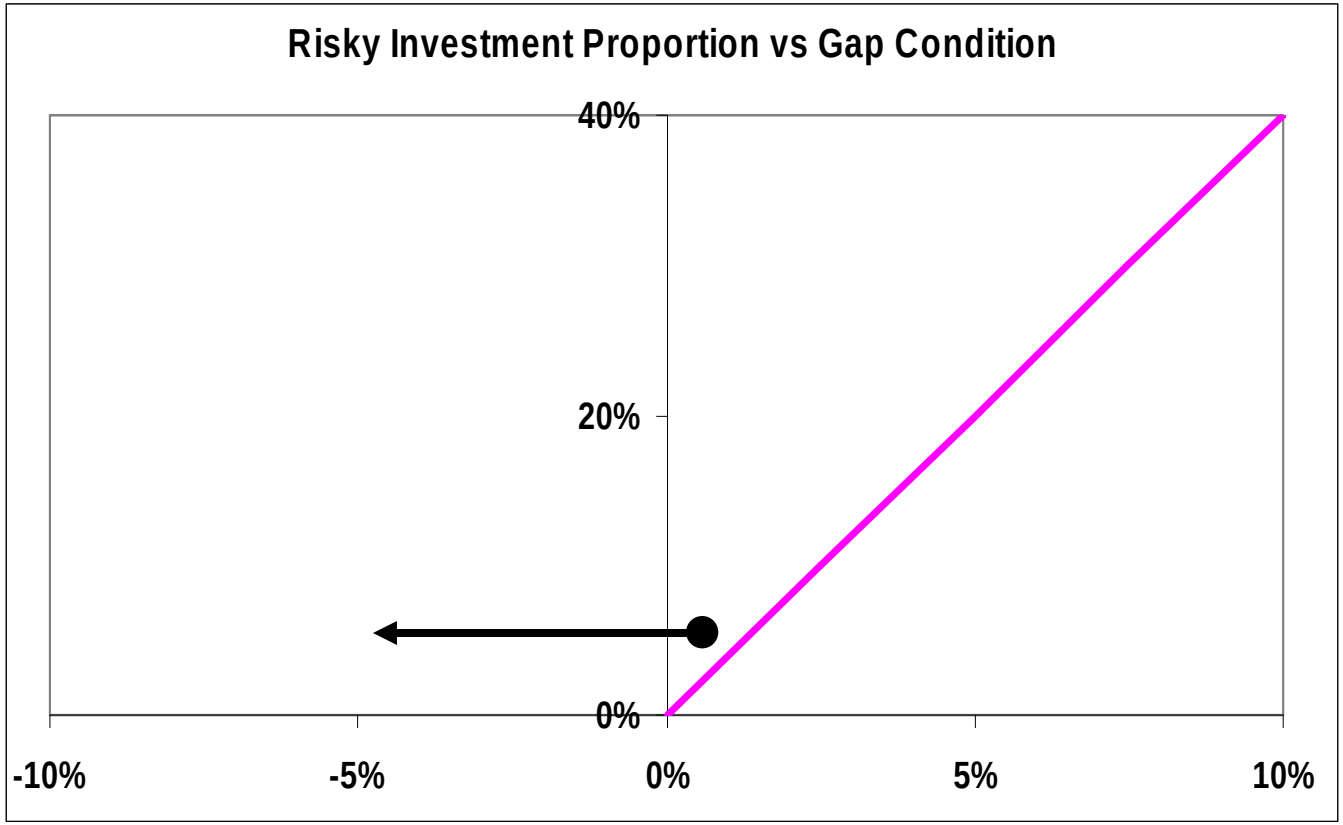
CPPI - Dynamic Re-balance

- **Switching between risky and risk free assets:**
  - **If *GC* is large (e.g. > 25% Of G_t), whole fund will be invested in risky assets, to fully benefit from market upside;**
  - **If *GC* is zero (i.e. $V_t = G_t$), whole fund will be invested in cash, to fulfil the promised guarantee;**
  - **Between the two extreme cases, a portion will be invested in risky assets, with the remaining in risk free asset. The asset re-balance is conducted dynamically.**

An Example CPPI Path



Gap Risk


Risky Investment Proportion vs Gap Condition
40%
20%
0%
-10%
-5%
0%
5%
10%

Hybrid Risks

- **Equity Linked GAO: pension & insurance products, equity + interest rate + correlation risks:**

$$N_T = N_0 \cdot \max(S_T - K, 0)$$

$$A_T = N_T \cdot \max(r_T, g)$$

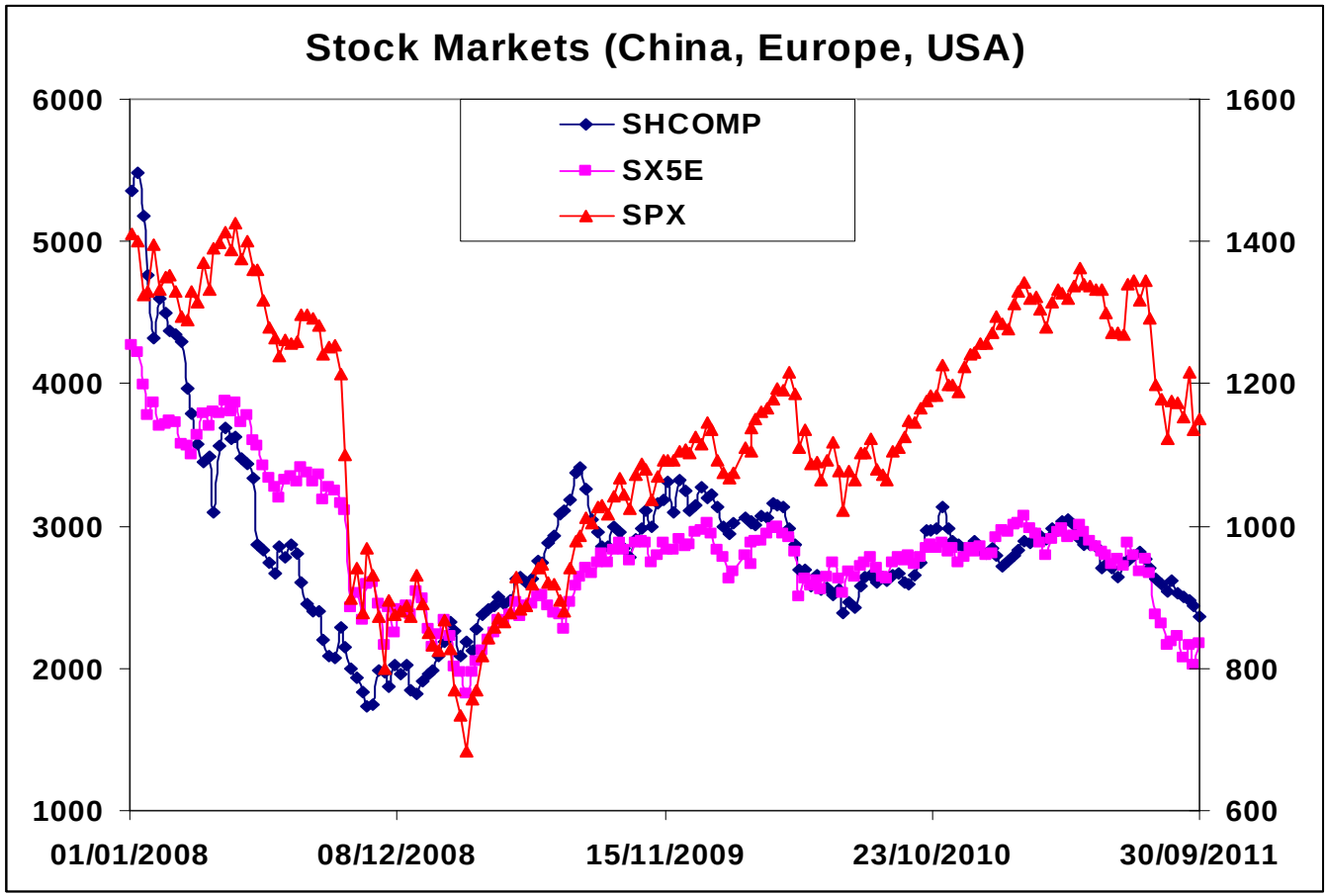
- **Equity & Debt Products: convertible bonds, equity + credit + correlation risks:**

$$\frac{\partial V_{CB}}{\partial t} + (r-q)\cdot S \cdot \frac{\partial V_{CB}}{\partial S} + \frac{1}{2}\sigma^2 \cdot S^2 \cdot \frac{\partial^2 V_{CB}}{\partial S^2} = r \cdot V_{CB} + c(S) \cdot V_B$$

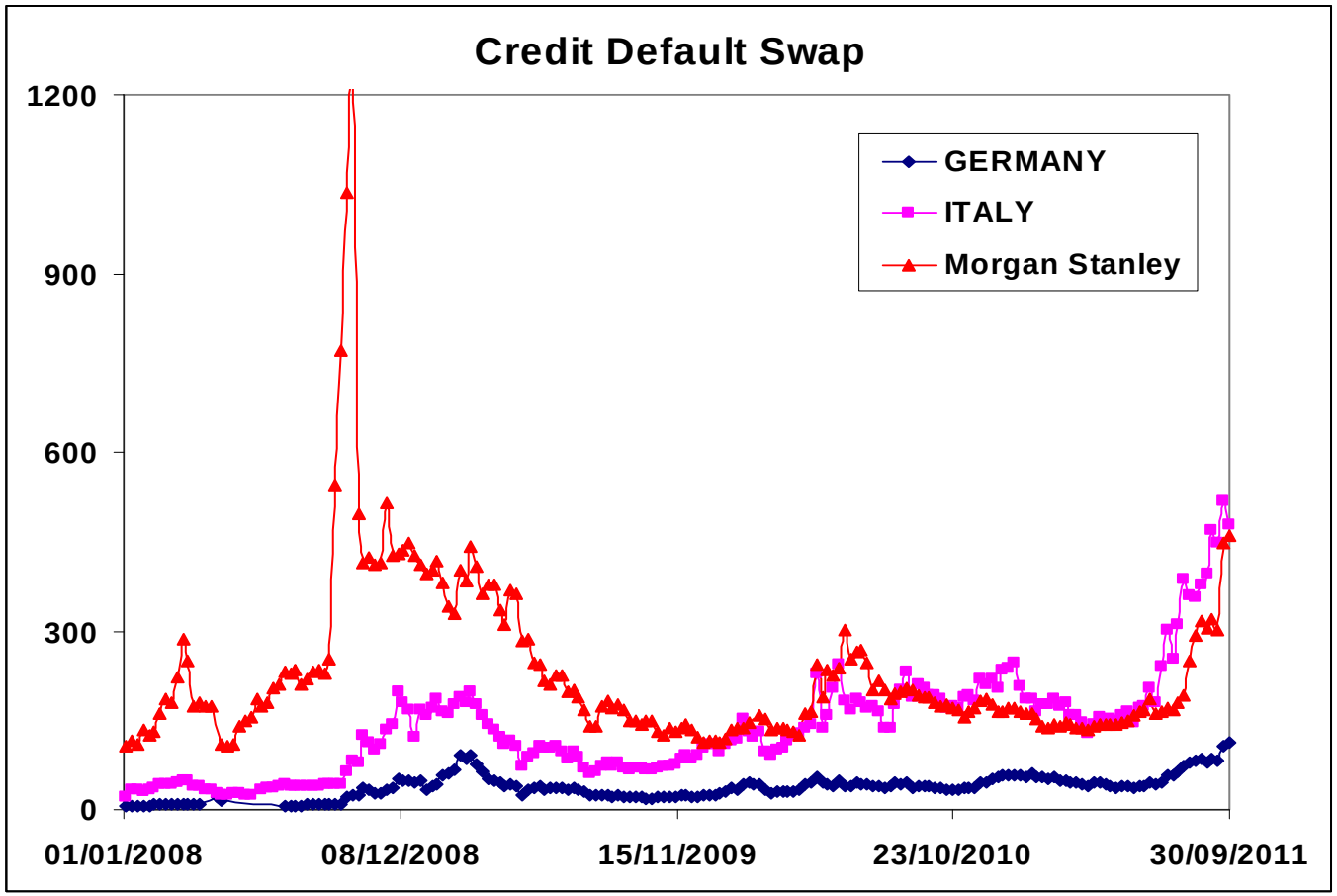
3. Correlation vs. Contagion

- **Stock markets correlation: global economy;**
- **Credit markets correlation: global economy;**
- **Stock/FX correlation: when markets are in distress;**
- **Credit/FX correlation: there have always been strong links between emerging markets FX and sovereign credits;**
- **Since credit crunch, many countries behave like emerging markets as far as credit is concerned.**

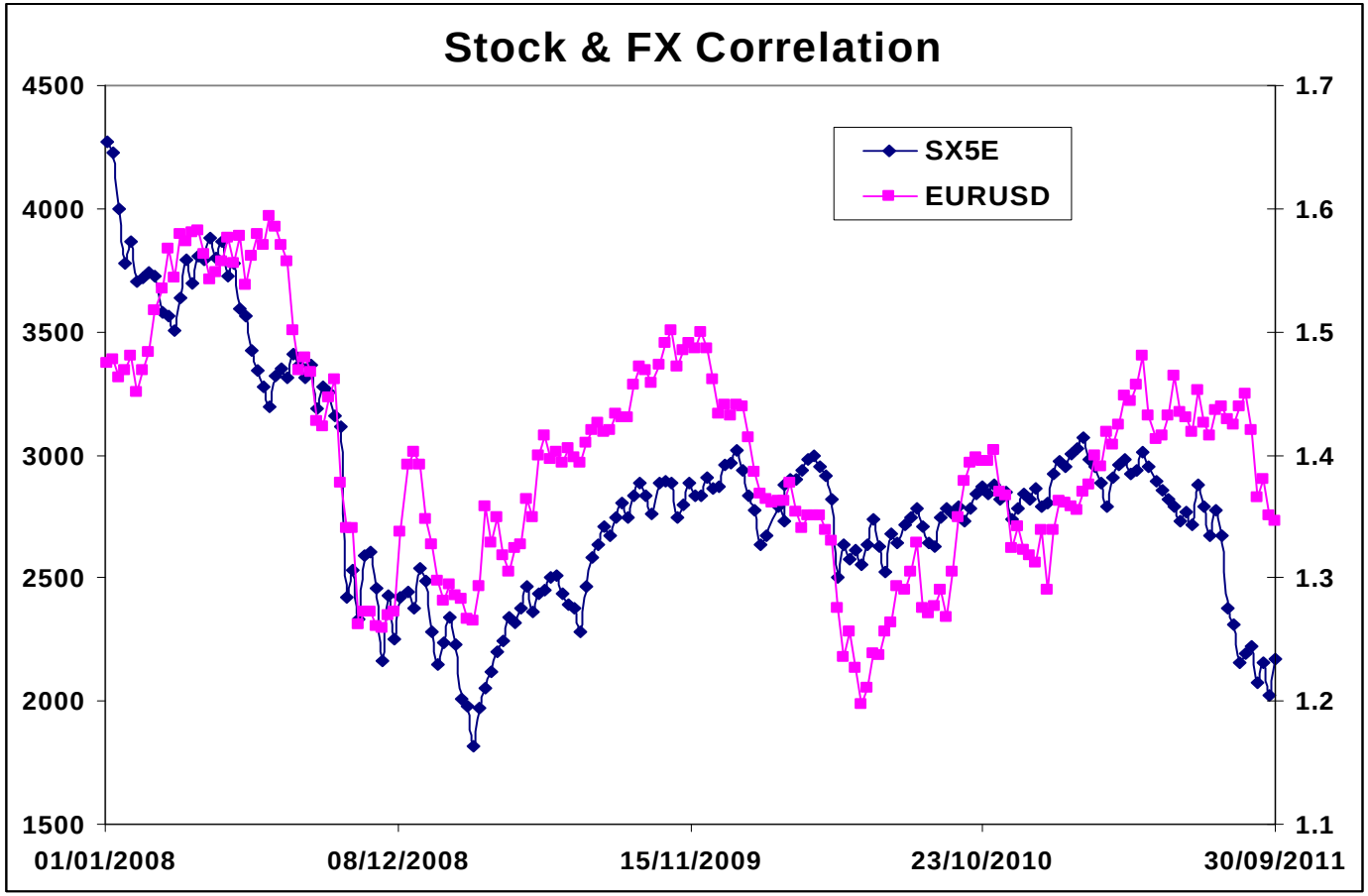
Stock Markets Correlation



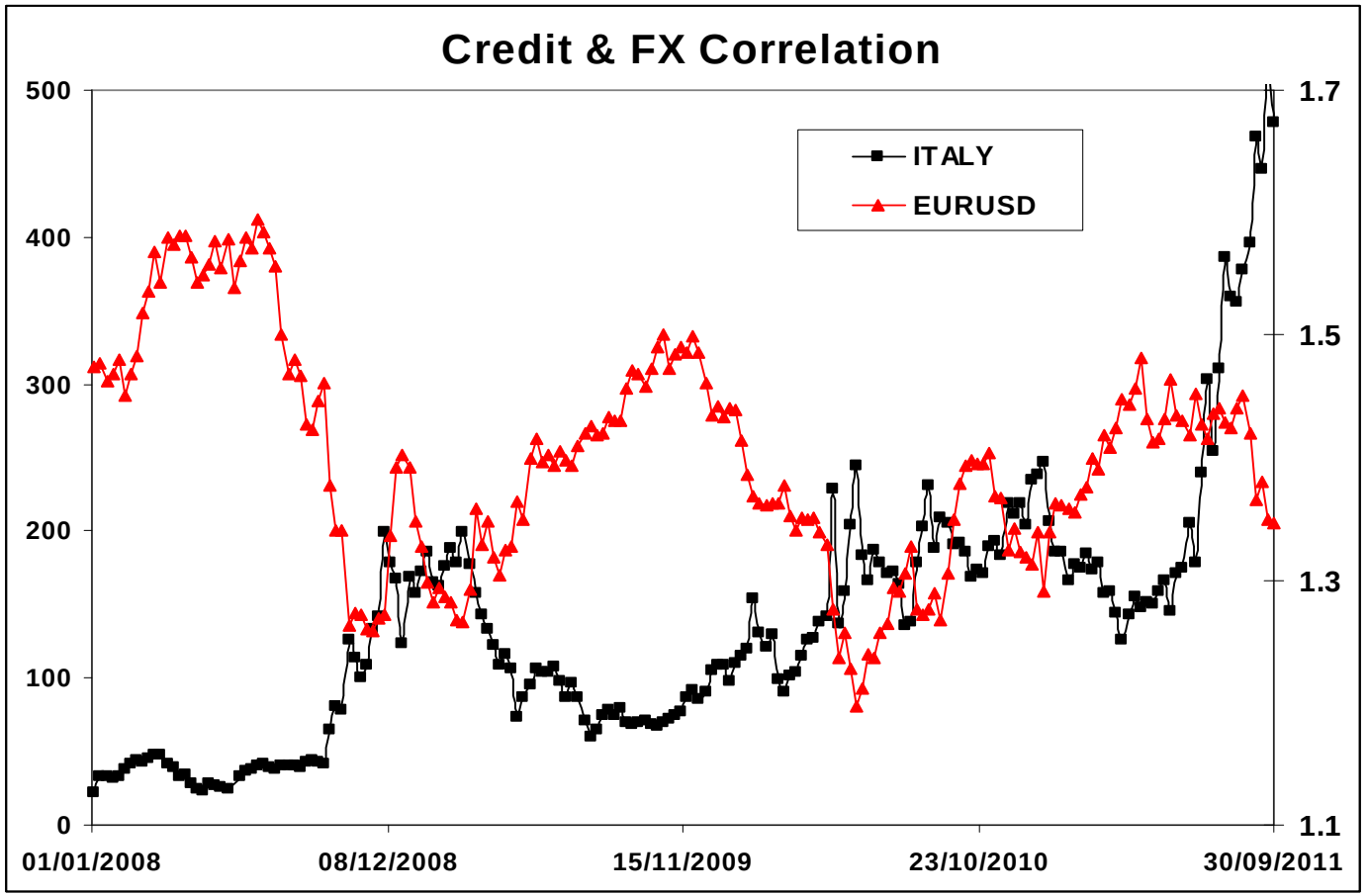
Credit Markets Correlation



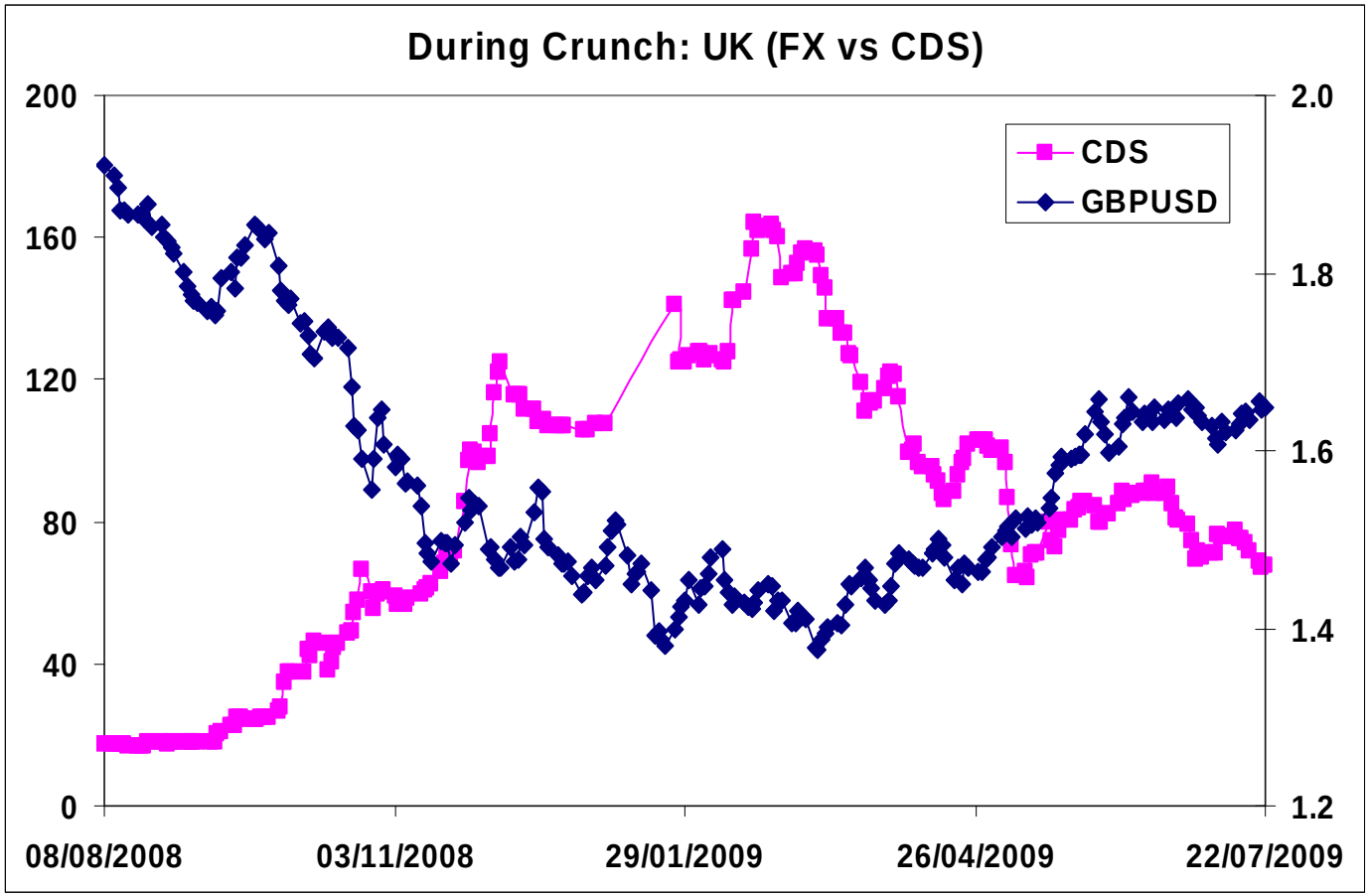
Stock & FX Correlation



Credit & FX Correlation



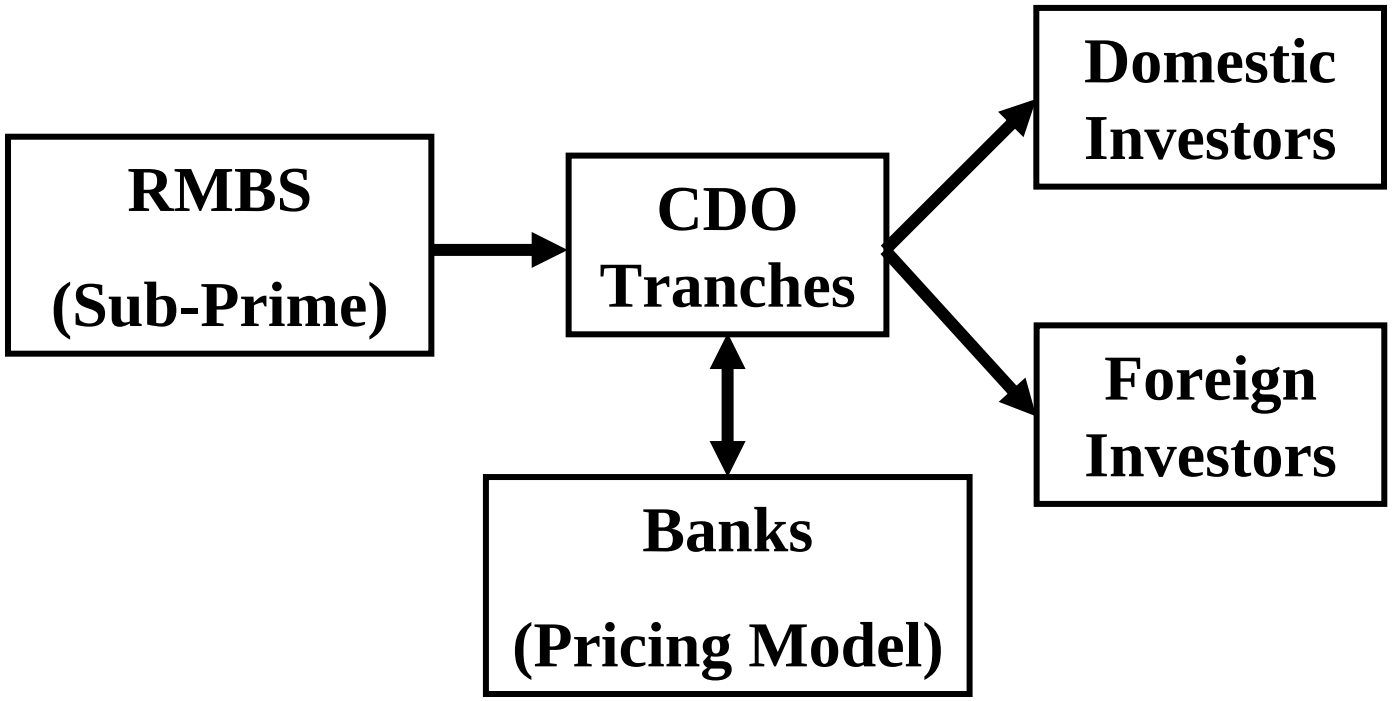
UK - CDS vs. FX (Credit Crunch)



What Happened In 2008?

- **Collateralized Debt Obligation (CDO): a pool of assets, with defined tranches of credit loss;**
  - **Each tranche can be sold to investors to suit their risk appetites;**
  - **The pool as a whole has no correlation risk, but pricing any single tranche CDO (STCDO) involves pricing in correlation risks;**
- **One needs to model joint default risks (correlation);**
- **Copula was used to model the joint default risks.**

Sub-Prime Distribution Mechanism



- All elements were in place! Ready to go!

CDO Lessons

- **There was a false sense of security – "we can now value CDOs". The world had since sold far too much of them!**

- **Simply because ones can value CDOs, it does NOT mean they can practically risk manage CDOs:**

  - **Hedging difficulties;**
  - **Over leveraging problems;**
  - **Correlation assumptions;**
  - **Liquidity issues;**
  - **Transparency & visibility issues;**
  - **Etc.**

What Is Happening Now (2011/2012)?

- **Instead of sub-prime mortgages in USA, it's now sovereign debts in Europe;**
- **Very high Debt to GDP Ratio: Greece (143%), Italy (119%), etc;**
- **In addition, GDP does not grow but decline;**
- **Faced with huge contagion risks, let's hope politians find answers very very soon;**
- **In fact, the contagion risks are surprisingly similar to basket correlation risks!**

Basket Options* Analogy

- **Basket correlation risks:**
  - **Cross Gamma: one underlying can affect the others;**
  - **Correlation risk: co-dependence can change;**
  - **Correlation skew risk: when markets fall, correlation shoots up;**
  - **Credit default risk: basket underlying can go bust;**

* **Qu, D. (2005), "Pricing Basket Options With Skew", Wilmott Magazine, July**

Contagion Risks

- **Contagion Risks: versus basket correlation risks:**
  - **Cross Gamma: one country can affect the others;**
  - **Correlation risk: co-dependence can change;**
  - **Correlation skew risk: when in crisis, countries' co-dependency becomes much stronger, hence increases chance of "getting worse together";**
  - **Credit default risk: a country can default.**

Basket Correlation ➔ Contagion

The World Is Like A Correlated Basket !!

Questions?