Credit Risk Modelling Before and After the Crisis

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Mini-Course on Credit Risk Modelling
Pisa, 28 March 2014

Talk Outline

1. Credit Derivatives
2. Pre-Crisis Pricing: multi-name credit products
3. Post-Crisis Pricing: credit, collateral and funding
4. Pricing Derivatives under CSA or CCP Clearing

Disclaimer

The opinions expressed in this work are solely those of the authors and do not represent in any way those of their current and past employers.

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Talk Outline

1. Credit Derivatives
   - Credit Default Swaps: a big bang
   - Risk-Neutral Pricing of CDS Contracts
   - Wrong-Way Risk and Gap Risk in CDS Contracts
2. Pre-Crisis Pricing: multi-name credit products
3. Post-Crisis Pricing: credit, collateral and funding
4. Pricing Derivatives under CSA or CCP Clearing

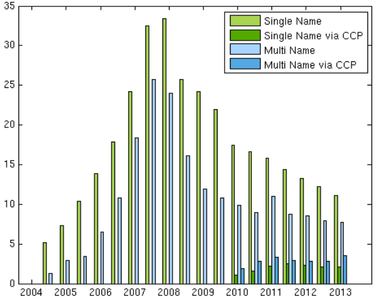
Credit Default Swaps – I

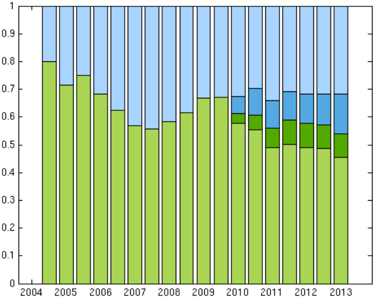
- A credit default swap (CDS) is a swap contract where the seller of the CDS will compensate the buyer in the event of a loan default or other credit event for a reference entity.
  - ⟶ The protection buyer of the CDS makes a series of fixed payments to the protection seller and, in exchange, receives a payoff if the loan defaults.
  - ⟶ It was invented by Blythe Masters from JP Morgan in 1994.
- When a credit event occurs the settlement of the CDS contracts can either be:
  - ⟶ physical, the protection seller pays the par value and receives a debt obligation of the reference entity.
  - ⟶ cash, the protection seller pays the difference between the par value and the market price of a debt obligation of the reference entity.

Credit Default Swaps – II

- In November 2012 the European Union introduced a set of rules to ban naked CDS.
  - ⟶ As a consequence emerging markets traded an increased number of CDS.
- From the Financial Times of 15 October 2013.
  - ⟶ "Synthetic CDOs or correlation desks were the guys that really drove the growth in single-name CDS."
  - ⟶ At the end of 2012, the notional amount of single-name CDS was halved from its peak in the first half of 2008.
  - ⟶ CDS indices have fared somewhat better attracting interest as both a trading and hedging product.
- From the Bloomberg Magazine of 30 Janaury 2014.
  - ⟶ What started as a simple hedging tool evolved into a playground for hedge funds and bank proprietary trading desks to speculate on debt, from corporate bonds to subprime mortgages

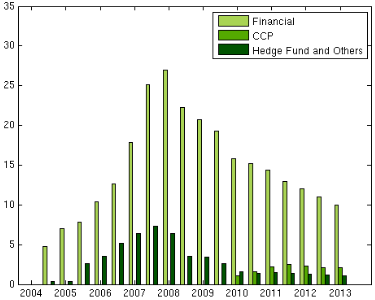
Credit Default Swaps – III

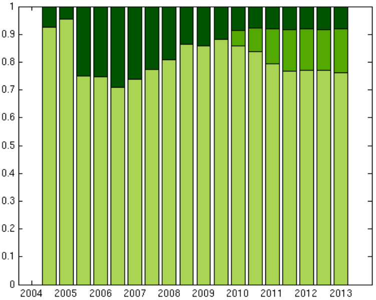




Total amount of single-name and multi-name credit default swaps on the market in trillion of dollars. Source Bank of International Settlements, OTC derivatives statistics (2013).

Credit Default Swaps – IV





Total amount of single-name credit default swaps on the market in trillion of dollars: bilateral contracts traded by financial institutions, by hedge funds and centrally cleared. Source Bank of International Settlements, OTC derivatives statistics (2013).

The CDS Big Bang – I

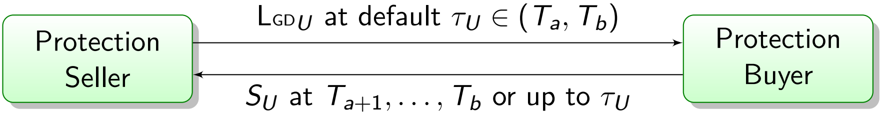
- The credit derivatives industry faces a Big Bang on April 2009 when ISDA implemented the new CDS protocol, including
  - ⟶ more consistency into the credit default swaps market by imposing a uniform procedure for settling CDS contracts when a company goes into default;
  - ⟶ more standardisation by introducing a set of possible values for the fixed payments for the contracts.
- Starting from 2009 CDS are quoted in term of an uprfont premium.
  - ⟶ The fixed payments can be equal to 100 or 500 bp depending of the quality of the credit.
  - ⟶ The recovery is also standardised to two possible values, again depending on the credit quality: 20% or 40%.
  - ⟶ See Beumee et al. (2009).

The CDS Big Bang – II

- A new Big Bang is coming?
- ISDA on 1 March 2013 declared that the restructuring of Greeks bonds did not constitute a credit event... but on 9 March 2013 it confirmed that a credit event had occurred.
  - ⟶ Although Greek CDS contracts will be settled in the weeks ahead, there remain concerns that CDS are flawed.
- The 2014 ISDA Credit Derivatives Definitions introduce several new terms, including:
  - ⟶ a new credit event triggered by a government-initiated bail-in;
  - ⟶ the ability to settle a credit event by delivery of assets into which sovereign debt is converted;
  - ⟶ the adoption of a standardized reference obligation across all market-standard CDS contracts.

CDS Payoff

- CDS are contracts that have been designed to offer protection $\text{L}_{\text{GD}U} := 1 - R_U$ against default of a reference name at τ_U in exchange for a periodic premium S_U.



- Thus, the coupon process for a receiver CDS is given by

$$
\begin{aligned}
d\pi_t^{\text{CDS}} \quad &:= \quad S_U \sum_{i=a+1}^{b} (\min\{T_i, \tau_U\} - T_{i-1}) 1_{\{\tau_U > T_{i-1}\}} \delta_t(T_i)\, dt \\
&- \quad \text{L}_{\text{GD}U} 1_{\{T_a < \tau < T_b\}} \delta_t(\tau_U)\, dt
\end{aligned}
$$

Market and Enlarged Filtrations – I

- How can we deal with the default event under the risk-neutral measure?
  - $\longrightarrow$ We need to describe the filtration to adopt to calculate the risk-neutral expectations.
- Market risks for CDS contracts arise from the uncertainty both in default probabilities and in the default time.
  - $\longrightarrow$ We could add interest-rates and recoveries as well.
- As a first step we introduce the market filtration $\mathcal{F}_t$ representing all the observable market quantities but the default event.

Market and Enlarged Filtrations – II

- Then, we define the enlarged filtration containing also the default monitoring.

  ⟶ See Bielecki and Rutkowski (2001) for details.

$$\mathcal{G}_t := \mathcal{F}_t \vee \sigma(\{\tau_U \leq u\} : u \leq t) \supseteq \mathcal{F}_t \quad (1)$$

- In the following, when we will deal with multiple names, we can generalize the above defintion by repeating the enlargement for each name.
- From the definition of $\mathcal{G}_t$, we obtain that any event in $\mathcal{G}_t$ has the form

$$\forall g_t \in \mathcal{G}_t \; \exists f_t \in \mathcal{F}_t : g_t \cap \{\tau_U > t\} = f_t \cap \{\tau_U > t\}$$

Market and Enlarged Filtrations – III

- Thus, for any $\mathcal{G}$-adapted process x_t we can introduce the pre-default $\mathcal{F}$-adapted process $\tilde{x}_t$ such that

$$1_{\{\tau_U > t\}} x_t = 1_{\{\tau_U > t\}} \tilde{x}_t \tag{2}$$

  by taking the expectation w.r.t. the market filtration, we get

$$\tilde{x}_t \, \mathbb{E}\left[1_{\{\tau_U > t\}} \,\middle|\, \mathcal{F}_t \right] = \mathbb{E}\left[1_{\{\tau_U > t\}} x_t \,\middle|\, \mathcal{F}_t \right]$$

- In particular, we can consider $x_t \doteq \mathbb{E}[1_{\{\tau_U > T\}} \phi | \mathcal{G}_t]$, where ϕ is a $\mathcal{F}_T$-integrable random variable, and we get

$$1_{\{\tau_U > t\}} \mathbb{E}\left[1_{\{\tau_U > T\}} \phi \,\middle|\, \mathcal{G}_t \right] = 1_{\{\tau_U > t\}} \frac{\mathbb{E}[\, \mathbb{Q}\{ \tau_U > T \,|\, \mathcal{F}_T \} \phi \,|\, \mathcal{F}_t]}{\mathbb{Q}\{ \tau_U > t \,|\, \mathcal{F}_t \}}$$

Market and Enlarged Filtrations – IV

Pricing Defaultable Claims – Jeulin and Yor (1978)

In a market with only one defaultable name we can calculate prices under market filtration, since we have

$$1_{\{\tau_U > t\}} \mathbb{E}\Big[1_{\{\tau_U > T\}} x_T \,\Big|\, \mathcal{G}_t \Big] = 1_{\{\tau_U > t\}} \frac{\mathbb{E}[\, \mathbb{Q}\{ \tau_U > T \,|\, \mathcal{F}_T \} \, \tilde{x}_T \,|\, \mathcal{F}_t]}{\mathbb{Q}\{ \tau_U > t \,|\, \mathcal{F}_t \}} \tag{3}$$

where x_t is a $\mathcal{G}$-adapted process, and $\tilde{x}_t$ is the corresponding pre-default process. In particular, we have also

$$1_{\{\tau_U > t\}} \mathbb{Q}\{ \tau_U > T \,|\, \mathcal{G}_t \} = 1_{\{\tau_U > t\}} \frac{\mathbb{Q}\{ \tau_U > T \,|\, \mathcal{F}_t \}}{\mathbb{Q}\{ \tau_U > t \,|\, \mathcal{F}_t \}} \tag{4}$$

- Thus, probabilities calculated w.r.t. filtration $\mathcal{F}_t$ and $\mathcal{G}_t$ are different.

CDS Pricing

- The risk-neutral price of a receiver CDS, without taking into account counterparty risk or funding costs, is given by

$$\begin{aligned} V_0^{\mathrm{CDS}} &:= \int_0^{T_b} \mathbb{E}\left[\, D(0,t)\, d\pi_t^{\mathrm{CDS}} \,|\, \mathcal{G}_0 \right] \\ &= S_U \sum_{i=a+1}^{b} \mathbb{E}\left[\, D(0,T_i)(\min\{T_i,\tau_U\} - T_{i-1}) 1_{\{\tau_U > T_{i-1}\}} \,|\, \mathcal{G}_0 \right] \\ &- \mathbb{E}\left[\, D(0,\tau) \mathrm{L}{\scriptstyle\mathrm{GD}}_U 1_{\{T_a < \tau_U < T_b\}} \,|\, \mathcal{G}_0 \right] \end{aligned} \tag{5}$$

- If we approximate the payments on a continuous basis we can write a simpler expression

$$V_0^{\mathrm{CDS}} = \int_{T_a}^{T_b} \mathbb{E}\left[\, D(0,t) \left(S_U 1_{\{\tau_U > t\}}\, dt + \mathrm{L}{\scriptstyle\mathrm{GD}}_U\, d1_{\{\tau_U > t\}} \right) \,|\, \mathcal{G}_0 \right]$$

Bootstrapping the Survival Probabilities – I

- Survival probabilities can be bootstrapped from CDS quotes.
- Many approximations are required to avoid a model-dependent procedure.
  - $\longrightarrow$ Recovery rates are uncertain and difficult to estimate.
  - $\longrightarrow$ CDS contracts are collateralized, but counterparty risk is still relevant due to contagion effects.
  - $\longrightarrow$ If CDS contracts are cleared via a CCP, funding costs may alter the quotes.
  - $\longrightarrow$ Interest-rates are usually correlated to default probabilities, so that they may impact the quotes as well.
- Moreover, the default event may be poorly defined as the recent Greece case shown.
- Yet, CDS are still the best candidate for a bootstrap procedure.
  - $\longrightarrow$ Rate agencies quotes default probabilities under historical measure in term of rating classes.

Bootstrapping the Survival Probabilities – II

- In the practice CDS are quoted with a deterministic recovery rate.
- Moreover, the analysis of Brigo and Alfonsi (2005) shows that we can safely assume independence of default probabilities from interest-rates when pricing CDS.
- Thus, since $\mathbb{Q}\{\,\tau_U > T \,|\, \mathcal{G}_0\,\} = \mathbb{Q}\{\,\tau_U > T \,|\, \mathcal{F}_0\,\}$, we can write

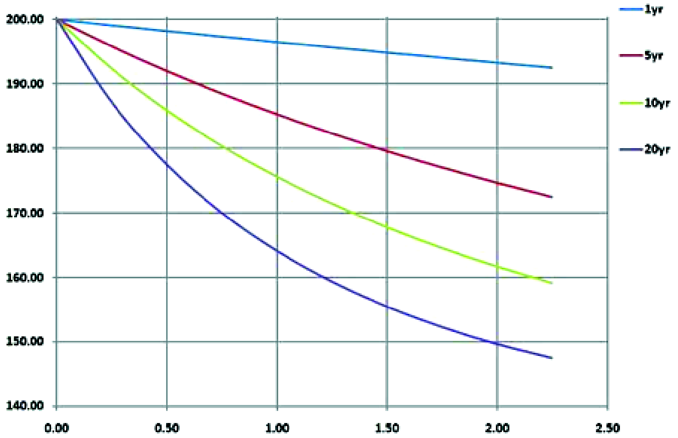
$$V_0^{\mathrm{CDS}} \doteq \int_{T_a}^{T_b} P_0(t) \left(S_U \, \mathbb{Q}\{\,\tau_U > t \,|\, \mathcal{F}_0\,\}\, dt + \mathrm{L{\scriptsize GD}}_U \, d\mathbb{Q}\{\,\tau_U > t \,|\, \mathcal{F}_0\,\} \right)$$

  and we can bootstrap the survival term structure as given by

$$T \mapsto \mathbb{Q}\{\,\tau_U > T \,|\, \mathcal{F}_0\,\}$$

- What happens if the protection seller defaults? Should we add a counterparty valuation adjustment?

Bootstrapping the Survival Probabilities – III



Change of par CDS spread for different maturities versus Clayton copula parameter. Details in Fujii and Takahashi (2011).

Counterparty Risk and Contagion Effects – I

- CDS contracts are collateralized on a daily basis to match their mark-to-market value.

  $\longrightarrow$ The collateral account is accrued at over-night rate e_t.

- This is the same collateralization policy used for interest-rate derivatives to remove almost all counterparty risk.

  $\longrightarrow$ Yet, this is not enough for credit derivatives.

- Counterparty risk happens when on default event the surviving party suffers a loss since some future cash flows are not wholly redeemed.

$$1_{\{\tau_C>t\}}\mathrm{CVA}_t^{\mathrm{CDS}} := -1_{\{\tau_C>t\}}\mathrm{L{\scriptstyle GD}}_C\int_t^{T_b} du P_t(u)\,\mathbb{E}\Big[\,\delta_u(\tau_C)\left(V_u^{\mathrm{CDS}} - C_{u^-}^{\mathrm{CDS}}\right)^+ \,|\,\mathcal{G}_t\Big]$$

where C_{u^-} is the collateral account evaluated just before time u, and τ_C is the countrparty default time.

Counterparty Risk and Contagion Effects – II

- In case of daily collateralization we can approximate $C_u^{\text{CDS}} \approx V_u^{\text{CDS}}$, and we get

$$1_{\{\tau_C > t\}} \text{CVA}_t^{\text{CDS}} := -1_{\{\tau_C > t\}} \text{L}_{\text{GD}C} \int_t^{T_b} du \, P_t(u) \, \mathbb{E}\Big[\delta_u(\tau_C) \left(\Delta V_u^{\text{CDS}} \right)^+ \, | \, \mathcal{G}_t \Big]$$

  where ΔV_u^{CDS} is different from zero if V_u^{CDS} jumps at u, namely at the default time.
- The CDS price in our approximation depends only on default probabilities, so that it may jump only if

$$\mathbb{Q}\{ \tau_U > t \, | \, \mathcal{G}_{\tau_C} \} \neq \mathbb{Q}\{ \tau_U > t \, | \, \mathcal{G}_{\tau_C -} \}$$

- This happens when there is a correlation between the reference name and the counterparty.

  $\longrightarrow$ A similar argument holds when both the parties can default.

Contagion Effects in CDS Pricing – I

- What happens to the reference name default probabilities after the counterparty default event?
  - ⟶ We assume that the counterparty defaults at time $t < u$, while the reference name defaults after u.
  - ⟶ After time t we have a single-name market.
- Thus, given a $\mathcal{G}$-adapted process x_t, we can write

$$1_{\{\tau_U > u\}} 1_{\{\tau_C = t\}} x_u = 1_{\{\tau_U > u\}} 1_{\{\tau_C = t\}} \tilde{x}_u$$

where $\tilde{x}_t$ is the corresponding $\mathcal{F}$-adapted pre-default process.

Contagion Effects in CDS Pricing – II

- If we take expecations w.r.t. the market filtration we obtain

$$\tilde{x}_u \, \partial_v \mathbb{Q}\{ \tau_U > u, \tau_C > v \mid \mathcal{F}_u \}|_{v=t} = \mathbb{E}\left[1_{\{\tau_U > u\}} 1_{\{\tau_C = t\}} x_u \mid \mathcal{F}_u \right]$$

- Again we can consider the case $x_t \doteq \mathbb{E}[1_{\{\tau_U > T\}} \phi | \mathcal{G}_t]$, where ϕ is a $\mathcal{F}_T$-integrable random variable, to get the generalization of the filtration switching theorem to a two-name market.

$$1_{\{\tau_U > u\}} 1_{\{\tau_C = t\}} \mathbb{E}\left[1_{\{\tau_U > T\}} x_T \mid \mathcal{G}_u \right] =$$

$$1_{\{\tau_U > u\}} 1_{\{\tau_C = t\}} \frac{\mathbb{E}[\, \partial_v \mathbb{Q}\{ \tau_U > T, \tau_C > v \mid \mathcal{F}_u \}|_{v=t} \, \tilde{x}_T \mid \mathcal{F}_u]}{\partial_v \mathbb{Q}\{ \tau_U > u, \tau_C > v \mid \mathcal{F}_u \}|_{v=t}}$$

Contagion Effects in CDS Pricing – III

Two-Name Default Probabilities – Schönbucher and Schubert (2001)

In a market with two defaultable names before any default event the default probabilities are given by

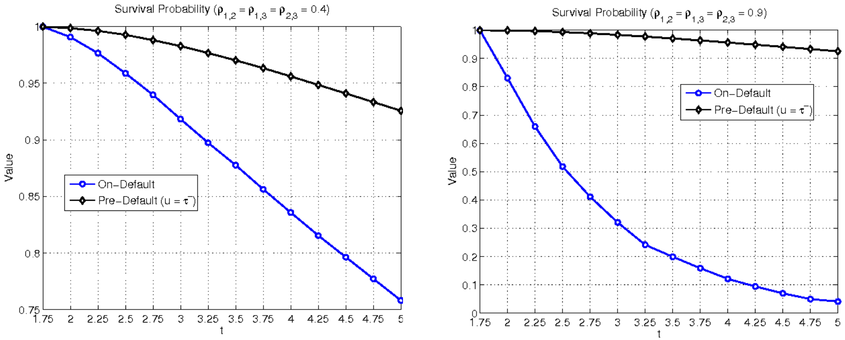
$$1_{\{\tau_U > t\}} 1_{\{\tau_C > t\}} \mathbb{Q}\{\, \tau_U > T \mid \mathcal{G}_t \,\} = 1_{\{\tau_U > t\}} 1_{\{\tau_C > t\}} \frac{\mathbb{Q}\{\, \tau_U > T, \tau_C > t \mid \mathcal{F}_t \,\}}{\mathbb{Q}\{\, \tau_U > t, \tau_C > t \mid \mathcal{F}_t \,\}} \quad (6)$$

while on a default event the probabilities jump to

$$1_{\{\tau_U > \tau_C\}} \mathbb{Q}\{\, \tau_U > T \mid \mathcal{G}_{\tau_C} \,\} = 1_{\{\tau_U > \tau_C\}} \lim_{t \downarrow \tau_C} \frac{\partial_v \mathbb{Q}\{\, \tau_U > T, \tau_C > v \mid \mathcal{F}_t \,\}|_{v=t}}{\partial_v \mathbb{Q}\{\, \tau_U > t, \tau_C > v \mid \mathcal{F}_t \,\}|_{v=t}} \quad (7)$$

- The first part of the theorem can be obtained from the single-name case by defining the pre-default process w.r.t. the first default event.
- The theorem can be generalized to many names.

Default Probabilities On-Default Jump



Comparison between on-default survival probabilities and pre-default survival probabilities at 1.75 years. Left panel: Gaussian copula parameter is 40%. Right panel: Gaussian copula parameter is 40%. Details in Brigo, Capponi, Pallavicini (2011).

CDS Instantaneous Gap Risk

CDS Instantaneous Gap Risk – Brigo, Capponi, Pallavicini (2011)

Collateralization cannot remove counterparty risk from a CDS.

$$1_{\{\tau_C>t\}}\mathrm{CVA}_t^{\mathrm{CDS}} = -1_{\{\tau_C>t\}}\mathrm{L}_{\mathrm{GD}_C}\int_t^{T_b} du\, P_t(u)\,\mathbb{E}\Big[\,\delta_u(\tau_C)\,\big(\Delta V_u^{\mathrm{CDS}}\big)^+ \,|\, \mathcal{G}_t\Big] \tag{8}$$

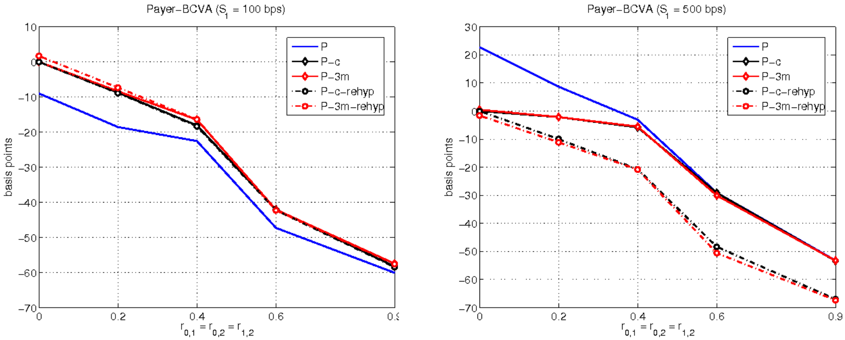
$$1_{\{\tau_U>t\}} V_t^{\mathrm{CDS}} = 1_{\{\tau_U>t\}}\int_t^{T_b} du\, P_t(u)\,(S_U\,\mathbb{Q}\{\,\tau_U>u \,|\, \mathcal{G}_t\,\} + \mathrm{L}_{\mathrm{GD}_U}\, d\mathbb{Q}\{\,\tau_U>u \,|\, \mathcal{G}_t\,\})$$

$$1_{\{\tau_U>\tau_C\}}\mathbb{Q}\{\,\tau_U>T \,|\, \mathcal{G}_{\tau_C-}\,\} = 1_{\{\tau_U>\tau_C\}}\lim_{t\uparrow\tau_C}\frac{\mathbb{Q}\{\,\tau_U>T,\tau_C>t \,|\, \mathcal{F}_t\,\}}{\mathbb{Q}\{\,\tau_U>t,\tau_C>t \,|\, \mathcal{F}_t\,\}}$$

$$1_{\{\tau_U>\tau_C\}}\mathbb{Q}\{\,\tau_U>T \,|\, \mathcal{G}_{\tau_C}\,\} = 1_{\{\tau_U>\tau_C\}}\lim_{t\downarrow\tau_C}\frac{\partial_v\mathbb{Q}\{\,\tau_U>T,\tau_C>v \,|\, \mathcal{F}_t\,\}|_{v=t}}{\partial_v\mathbb{Q}\{\,\tau_U>t,\tau_C>v \,|\, \mathcal{F}_t\,\}|_{v=t}}$$

- The theorem can be generalized to include the investor default event.

CVA and DVA for CDS Contracts



Bilateral credit adjustment, namely the algebraic sum of CVA and DVA, versus default correlation under different collateralization strategies for a five-year payer CDS contract. Left panel: the CDS spread is 100bp. Right panel: the CDS spread is 500bp. Details in Brigo, Capponi, Pallavicini (2011).

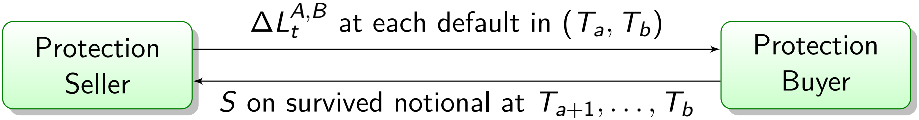
Talk Outline

An Introduction to CDOs – I

- In a CDO there are two parties, a protection buyer and a protection seller.
  - ⟶ Protection is bought (and sold) on a reference pool of M names.
  - ⟶ Most liquid CDOs (iTraxx or CDX) consider a pool of $M = 125$ names.
- The names may default, generating losses (L) to investors exposed to those names.
  - ⟶ Each time a name defaults the protection seller pays the protection buyer for the suffered loss.
- If the CDO is *tranched*, then only a portion of the loss of the portfolio between two percentages A and B is repayed.

$$L_t^{A,B} := \frac{M}{B-A}\left[\left(\frac{L_t}{M} - A\right)1_{\{A<\frac{L_t}{M}<B\}} + (B-A)\,1_{\{\frac{L_t}{M}>B\}}\right]$$

An Introduction to CDOs – II



- Thus, if we approximate the payments on a continuous basis, the price of a receiver CDO tranche can be written as

$$V_0^{A,B} = \int_{T_a}^{T_b} \mathbb{E}\left[D(0,t) \left(S^{A,B}(1 - L_t^{A,B})\, dt - dL_t^{A,B} \right) \,\middle|\, \mathcal{G}_0 \right]$$

- As for CDS an upfront can be payed at contract inception.

An Introduction to CDOs – III

- Originally developed for the corporate debt markets, over time CDOs evolved to encompass various asset classes, such as
  - $\longrightarrow$ loans (CLO),
  - $\longrightarrow$ residential mortgage portfolios (RMBS),
  - $\longrightarrow$ commercial mortgages portfolios (CMBS), and on and on.
- For many of these CDOs, and especially RMBS, quite related to the asset class that triggered the crisis, the problem is in the data rather than in the models.
  - $\longrightarrow$ At times data for valuation in mortgages CDOs (RMBS and CDO of RMBS) can be distorted by fraud.
- Even bespoke corporate pools have no data from which to infer default "correlation" and dubious mapping methods are used.
- At times it is not even clear what is in the portfolio, e.g. from the offering circular of a huge RMBS (more than 300.000 mortgages)

Copula-Based Modelling – I

- Since tranched loss is a non-linear function of single-name losses, the tranche expectation will depend both on:
  1. marginal distributions of the single names' defaults, and on
  2. dependency (or with abuse of language "correlation") among different names' defaults.
- The complete description is either the whole multivariate distribution or the so-called *copula function* where marginal distributions have been standardized to uniform distributions.

$$F_X(x) := \mathbb{Q}\{ X \leq x \} \ , \quad F_Y(y) := \mathbb{Q}\{ Y \leq y \}$$

$$C(u,v) := \mathbb{Q}\left\{ X \leq F_X^{-1}(u), Y \leq F_Y^{-1}(v) \right\}$$

- Notice that copulas do not define a dynamics for default processes and the choice of a particular copula family is arbitrary: Gaussian, t-Student, Archimedean, Marshall-Olkin, ...

Copula-Based Modelling – II

- If we model the default probabilities of single names as the first default event of a Poisson process, we can write

$$\mathbb{E}[\,\tau_i \leq t \,|\, \mathcal{G}_0\,] = 1 - e^{-\Lambda_i(t)} \;, \quad \Lambda_i(t) := \int_0^t du\, \lambda_i(u)$$

  where $\lambda_i(t)$ is the default intensity of name i, which we assume to be deterministic.

- For each name i we write

$$\mathbb{E}[\,\tau_i \leq t \,|\, \mathcal{G}_0\,] \doteq \Phi(X_i) \;, \quad X_i \sim \mathcal{N}(0,1)$$

  where Φ is the normal cumulative distribution.

- Then, we introduce dependencies among default times by correlating the latent factors X_i.

  $\longrightarrow$ This is the Gaussian copula model.

The Gaussian Copula Model – I

- The dependence of the tranche on “correlation” is crucial.

  $\longrightarrow$ The market assumes a Gaussian copula connecting the defaults of the 125 names, parametrized by a correlation matrix with

  $$125 \cdot 124/2 = 7750 \;\; \text{entries.}$$

  $\longrightarrow$ However, when looking at a tranche:

  $$7750 \;\; \text{parameters} \;\; \longrightarrow \;\; 1 \;\; \text{parameter.}$$

- *“The most dangerous part is when people believe everything coming out of it.”* [David Li, 2005, Wall Street Journal]

  $\longrightarrow$ Investors who put too much trust in it or do not understand all its subtleties may think they have eliminated their risks when they have not.

The Gaussian Copula Model – II

- Hence, in the one-factor version of the Gaussian copula model we have one common latent factor for all X, so that

$$L_t = \mathrm{L_{GD}} \int dx\, \varphi(x) \prod_{i=1}^{125} \Phi\left(\frac{\Phi^{-1}\big(1 - e^{-\Lambda_i(T)}\big) - \sqrt{\rho}\, x}{\sqrt{1-\rho}} \right) \tag{9}$$

  where φ is the normal probability density, and $\mathrm{L_{GD}}$ is the typical level for the loss given default.

- The model is calibrated implying the (compound) correlations from the tranche quotes.
  - ⟶ If at a given time the 3%–6% tranche for a CDO has a given implied correlation, the 6%–9% tranche for the same maturity will have a different one.
  - ⟶ The two tranches on the same pool are priced (and hedged) with two *inconsistent* loss distributions.
  - ⟶ Moreover, implying correlation could be unfeasible.

The Gaussian Copula Model – III

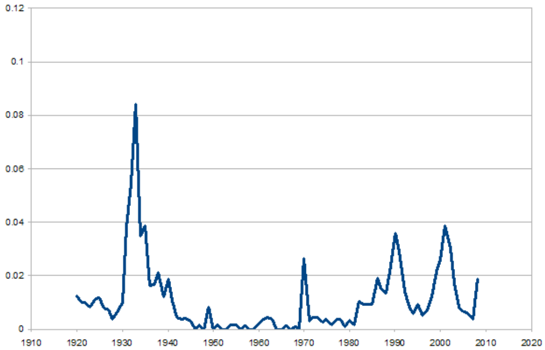
Beyond Copulas

- Alternative models for implied correlations, based on different parametrizations, were proposed.
  - $\longrightarrow$ For instance the base correlation model extended with stochastic correllation as in Amraoui and Hitier (2008).
- There are several publications that appeared pre-crisis and that questioned the Gaussian copula and implied correlations.
  - $\longrightarrow$ On the Wall Street Journal: "How a Formula Ignited Market That Burned Some Big Investors" (2005).
  - $\longrightarrow$ For further details see Torresetti, Brigo and Pallavicini (2006).
- Which are the ingredients missing in a copula-based model?

Default Clustering – I

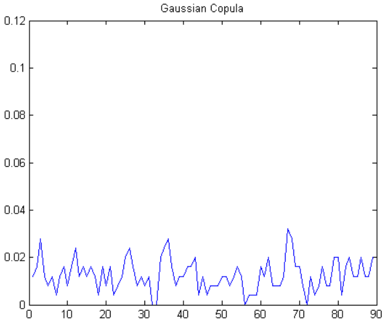
- In the recent history we observe many times cluster of default events.
  - ⟶ Thrifts in the early 90s at the height of the loan and deposit crisis.
  - ⟶ Airliners after 2001.
  - ⟶ Autos and financials more recently.
- From the September, 7 2008 to the October, 8 2008, we witnessed *seven* credit events: Fannie Mae, Freddie Mac, Lehman Brothers, Washington Mutual, Landsbanki, Glitnir, Kaupþing.
- Default clustering produces bumps in the right tail of loss distribution.
  - ⟶ Multi-modal loss distributions are present in some non-dynamical models, as in Hull and White (2006), Torresetti, Brigo and Pallavicini (2006), or Longstaff and Rajan (2008).
- Brigo, Pallavicini and Torresetti (2006,2007) propose default clustering with the GPL and GPCL dynamical models.
  - ⟶ The GPL model was the first model succeeding in calibrating all the quotes of iTraxx and CDX.

Default Clustering – II



Annual issuer-weighted global default rates by letter rating, Moody's 1920-2008.

Default Clustering – III



Simulation paths of the default rate for the Gaussian copula model (on the left) and the GPCL model (on the right) under the objective measure rescaled to match the average historical default rate from 1920 to 2008.

Default Clustering – IV

- Default clustering may be viewed as an extreme way of modelling the self-excitement of the loss process.
  - ⟶ A self-excited loss process means that one default increases the intensity of others.
  - ⟶ The collapse of Lehman Brothers brought the financial system near to a breakdown.
  - ⟶ Lehman was an important node within a network of derivative contracts: it sold CDS on a large number of firms and it was itself a reference entity in many other CDS.
  - ⟶ Its default triggered other insurance sellers into default, leaving the corresponding protection buyers with losses, etc. . .
- Errais, Giesecke and Goldberg (2006) introduce self-excitement effects to calibrate iTraxx and CDX quotes.

Generalized Poisson Loss Models – I

- We model the total number of defaults in the pool by time t as

$$Z_t := \sum_{j=1}^{n} \alpha_j Z_j(t) \tag{10}$$

  where α is a vector of positive integers, and Z are independent Poisson processes.

- If Z_j jumps there are as many defaults as the value of α_j.

  $\longrightarrow$ Just one default (idiosyncratic) if $\alpha_j = 1$, or the whole pool in one shot (total systemic risk) if $\alpha_j = M$, otherwise for intermediate values we have defaults of whole sectors.

Generalized Poisson Loss Models – II

- Modelling the counting process as a sum of Poisson processes may lead to an infinite number of defaults.

  ⟶ This approach is followed by Lindskog and McNeil (2003) to model insurance losses.

- A first solution (GPL) is modifying the counting process so that it does not exceed the number of names, by simply capping Z_t to M, regardless of cluster structures:

$$C_t \doteq \min(Z_t, M)$$

- That choice works at aggregate loss level, but it does not really go down towards single names' dynamics.

  ⟶ The aggregate loss is capped, but we cannot track which single name is jumping.

Generalized Poisson Loss Models – III

- A second solution (GPCL) is forcing clusters to jump only once and deduce single names defaults consistently.
- We introduce a set of independent Poisson processes $\widetilde{N}_s$ for each cluster s, and we define the indicator J_s as given by

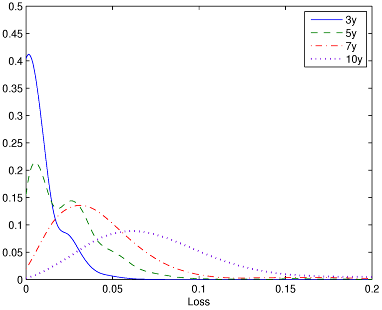
$$J_s(t) := \prod_{k \in s} \prod_{s' \ni k} 1_{\{\widetilde{N}_{s'}(t)=0\}}$$

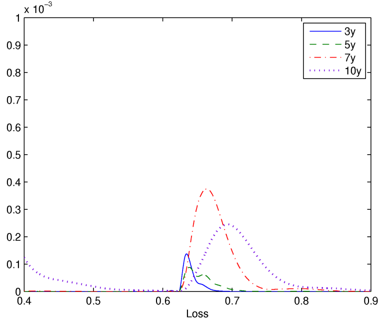
leading to the following single-name and multi-name dynamics

$$dN_k(t) = \sum_{s \ni k} J_s(t^-)\, d\widetilde{N}_s(t)\,, \quad dC_t = \sum_{j=1}^{n} \alpha_j \sum_{|s|=j} J_s(t^-)\, d\widetilde{N}_s(t) \qquad (11)$$

- That choice is a real top-down model, but it is combinatorially more complex.

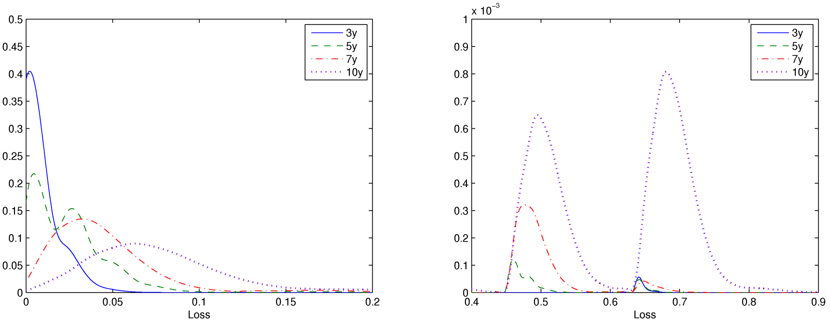
Implied Loss Distributions – I





Left panel: implied iTraxx loss distribution on 2 Oct 2006 by the GPL model.
Right panel: zoom of the right tail.

Implied Loss Distributions – II

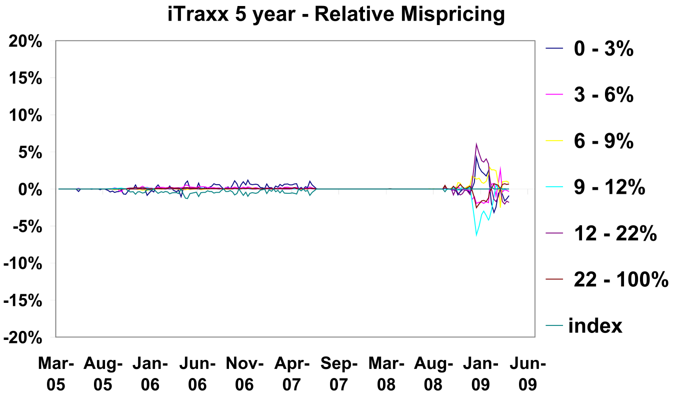


Left panel: implied iTraxx loss distribution on 2 Oct 2006 by the GPCL model.
Right panel: zoom of the right tail.

Calibration Results across the Crisis Period – I

- The market since 2008 has been quoting CDOs with prices assuming that the super-senior tranche would be impacted to a level impossible to reach with recoveries around 40%.
  - ⟶ Only huge losses affect super-senior tranche pricing: at least one fourth of the pool for iTraxx.
  - ⟶ We can assign a small (or a zero) recovery to extreme events (higher modes of GPL/GPCL model).
- In GPL/GPCL dynamic loss models recovery can be made a function of default rate C or portfolio loss L, see Brigo Pallavicini and Torresetti (2007) for more discussion.
- A simple approach is assigning a zero recovery rate to the systemic event, corresponding to $\alpha_n = M$ mode, while $R = 40\%$ for the other events.
  - ⟶ See Brigo, Pallavicini and Torresetti (2009,2010).

Calibration Results across the Crisis Period – II

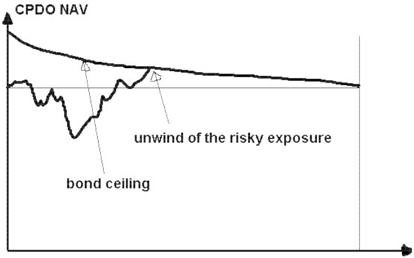


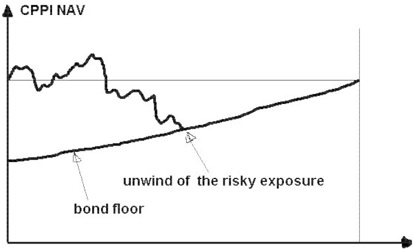
Calibration relative mispricing for CDX for all tranches and maturities throughout the sample ranging from March 2005 to June 2009. See Brigo, Pallavicini and Torresetti (2010).

Constant Proportion Debt Obligations – I

- A constant proportion debt obligation (CPDO) is a bond paying a spread over Libor, financed by a strategy that sells unfunded leveraged protection on a credit index trying to exploit the mean reverting properties of credit spread.
  - $\longrightarrow$ When the spread widens, and thus the strategy incurs a loss, the CPDO strategy increases the bet.
  - $\longrightarrow$ This is the opposite of constant proportion portfolio insurances (CPPI) where widening the spread the strategy reduces the leverage.
  - $\longrightarrow$ The minimum return on capital of a CPPI is 0% whereas for a CPDO can be -100%.
- Before the crisis agencies used to rate CPDOs as high rating notes.
- As soon as the net asset value (NAV) of the strategy is sufficient to guarantee the payment of the remaining fees, coupons and principal, the risky exposure is completely unwound.

Constant Proportion Debt Obligations – II





CPDOs and CPPIs have a very different NAV structure. CPDOs are limited from above by a bond ceiling, while CPPIs are limited from below by a bond floor.

Constant Proportion Debt Obligations – III

- CPDOs can be considered the latest and most extravagant of the structured credit products that arrived at the end of a prolonged boom in the credit markets.
- The anecdotical justification of their existence is to allow institutional investors to take advantage of the mean reverting nature of credit spreads through a mechanic trading strategy.
- It might be argued that it would be strange for an institutional investor to take a leveraged long exposure to credit on the peak of the credit market.
- Here, we propose a new rating model for CPDOs in order to incorporate a more realistic loss distribution showing a multi-modal shape, which, in turn, is linked to default possibilities for clusters (possibly sectors) of names in the economy.

Default Rates and CPDO Rating – I

- For CPDO rating we use the GPCL model to estimate the loss distribution in the objective measure preserving the multi-modal features that the model predicts in the risk-neutral measure.

  $\longrightarrow$ Here, we follow Torresetti and Pallavicini (2007,2010).
- Furthermore, since we are not interested in single-name dynamics we approximate cluster dynamics by considering cluster default-intensity depending only on cluster size (homogeneous pool assumption).
- In order to reduce the number of model parameters we consider some of them to be the same across calibration dates as in Longstaff and Rajan (2008).
- We can derive the aggregated pool-loss distribution in the objective measure by rescaling the risk-neutral loss distribution to match the probability of default of the underlying pool of names in between roll dates, which is obtained from Moody's rating transition matrix.

Default Rates and CPDO Rating – II

Series	Roll Date	0%-3%	3%-6%	6%-9%	9%-12%	12%-22%
iTraxx S1	20-Mar-2004	1.7	0.8	1.9	1.1	0.8
iTraxx S2	20-Sep-2004	0.4	0.5	0.8	2.0	1.1
iTraxx S3	20-Mar-2005	5.2	6.0	1.9	1.4	1.1
iTraxx S4	20-Sep-2005	1.6	1.6	1.5	0.8	0.9
iTraxx S5	20-Mar-2006	3.0	3.2	1.6	1.4	0.6
iTraxx S6	20-Sep-2006	1.3	1.9	1.9	1.8	0.6
iTraxx S7	20-Mar-2007	3.2	3.1	3.6	3.8	1.5

Average mispricing error in bid-ask spread units for the iTraxx series. The averages are calculated on all weekly market data when the series were on-the-run.

Default Rates and CPDO Rating – III

		copula		GPCL	
	Roll-Down Benefit	**3%**	**0%**	**3%**	**0%**
Default Rate	**Standard**	1.12%	3.52%	2.04%	7.16%
	Stressed	2.24%	9.76%	4.08%	11.32%
Loss Rate	**Standard**	0.99%	2.37%	2.36%	6.52%
	Stressed	2.21%	8.51%	4.79%	10.44%
CPDO Rating	**Standard**	AA	A-	A+	BBB-
	Stressed	A+	BBB-	BBB+	BBB-

CPDO average default-rate, loss-rate and rating (according to the probability of default estimated by S&P) for different values of the roll-down benefit and scenarios for index spread dynamics under objective measure. Details on Torresetti and Pallavicini (2007,2010).

Talk Outline

The Impact of Credit and Liquidity Risks – I

- After the crisis of 2007 derivative pricing cannot disregard credit and liquidity risks any longer.
  - ⟶ Cash flows are always risky since any counterparty can default.
  - ⟶ Cash and risky assets can be traded only in limited quantities.
- As a result the price of derivative contracts is now depending on
  - ⟶ the cash flows exchanged when the contract is early terminated, because of the default event of one the two parties, and on
  - ⟶ the cash flows exchanged to implement the collateral and funding procedures.
- Furthermore, the collateralization of hedging instruments, and any additional fee required to trade them, should be included in the pricing equations.
- See for details Pallavicini, Perini and Brigo (2011,2012), Crépey (2011), Bielecki and Rutkowski (2013).

The Impact of Credit and Liquidity Risks – II

- The impact on the pricing equations of these additional terms results in changing both the discount factors and the growth rates of underlying risk factors.
  - $\longrightarrow$ Under the assumption of funding and hedging in continuous time we obtain that the pricing equations do not depend on the risk-free rate.
- An important consequence is that the price of a derivative does depend in such framework on the collateral, funding and hedging procedures we are using.
- Multiple-curve frameworks and OIS-discounting practices are consequences of the above scenario.
- See for details Moreni and Pallavicini (2010,2013), Crépey, Grbac and Ngor (2012), Filipovic and Trolle (2012), Pallavicini and Brigo (2013).

The Multiple-Curve Framework

- Since the price of a derivative depends on the choice of collateral, funding and hedging procedures, we must differentiate bootstrap/calibration instruments according to these choices.
  - ⟶ When we calibrate our pricing models, as a first step, we have to choose quotes of instruments with the same set of collateral, funding and hedging procedures.
- We are not able to make prices for other choices, unless we properly adjust the discount factors and the growth rates.
  - ⟶ In general, such adjustments will be model dependent.
  - ⟶ As an example consider to calibrate an interest-rate model to OTC collateralized IRS, then to price either a non-collateralized IRS closed with a corporate or an IRS cleared via a centralized counterparty (CCP).
- See for details Cont, Mondescu, and Yu (2011), Pallavicini and Brigo (2013).

Modelling Funding Rates

- Funding rates depend on the investor credit quality, but also on the funding policy of the Treasury, which in turn depends on the business policy of the Bank, and it may change in time.
- Thus, it is difficult to model funding rates directly.
  - ⟶ A term structure of funding rates is published by the Treasury according to its policy.
  - ⟶ Yet, the option market (e.g. contingent funding derivatives) is missing.
- On the other hand, we can use proxies, which can be linked to the Bank credit quality (bonds) and to the collateral portfolio (re-hypothecation).
  - ⟶ Libor rates fail to be good proxies when the main source of funding is not represented by unsecured deposits.
- See for details Pallavicini, Perini and Brigo (2011), Castagna and Fede (2013).

Borrowing and Lending Rates

- Moreover, we can choose different proxies for borrowing and lending rates, depending on the definition of funding netting sets and on the policies adopted by the Treasury.
- A direct consequence of introducing differential rates is that short and long positions have different replication prices.
  - $\longrightarrow$ The market price of the derivative will lay within the bid-ask spread formed by such replication prices.
- See for details Bergman (1995), Peng (2003), Pallavicini, Perini and Brigo (2012), Mercurio (2013).

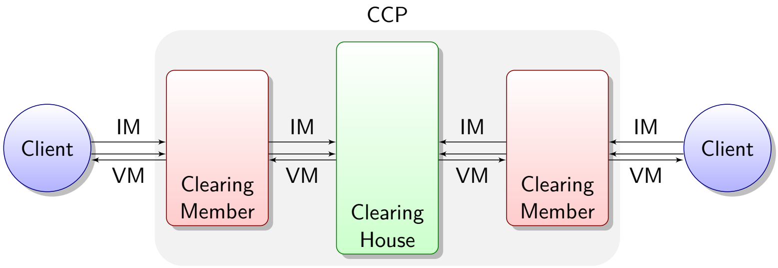
Collateralization and Counterparty Credit Risk

- The growing attention on counterparty credit risk is transforming OTC derivatives money markets:
  - $\longrightarrow$ an increasing number of derivative contracts is cleared by CCPs, while
  - $\longrightarrow$ most of the remaining contracts are traded under collateralization.
- Both cleared and CSA deals require collateral posting, along with its remuneration.
- Collateralized bilateral trades are regulated by ISDA documentation, known as Credit Support Annex (CSA).
- Centralized clearing is regulated by the contractual rules desribed by each CCP documentation.

Centralized Clearing – I

- Centralized counterparties are commercial entities that, ideally, would interpose themselves between the two parties in a trade.
  - ⟶ More specifically, a CCP acts as a market participant who is taking the risk of the counterparty default and ensures that the payments are performed even in case of default.
- To achieve this an initial bilateral trade is split into two trades, with the CCP standing in between the parties (clients).
  - ⟶ In practice, the counterparties operate with the CCP by means of intermediate clearing members.
- CCPs will reduce risk in many cases but are not a panacea.
  - ⟶ They require daily margining in an over-collateralization regime to account for wrong-way risk and gap-risk.
  - ⟶ Clearing members may default and their replacement may lead to additional costs.
  - ⟶ The CCP itself may default.

Centralized Clearing – II

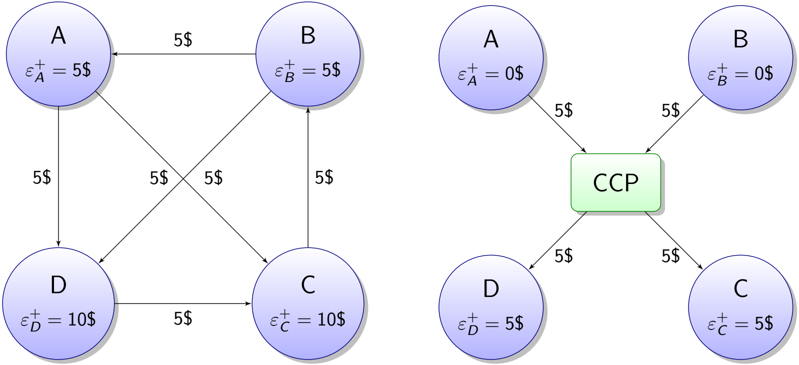


- There are no more direct obligations between the two orginal clients.
- Each party will post collateral margins: variation and intial margins.
- Variation margin can be re-hypothecated, while initial margin is segregated.

Centralized Clearing – III

- CCPs are usually highly capitalized, see Rhode (2011).
  - ⟶ Initial margin means clearing members are always over-collateralized.
  - ⟶ The TABB Group says extra collateral could be about 2 $ Trillion.
- CCPs default are to be kept in mind, see Piron (2012).
  - ⟶ Defaulted CCPs include: 1974, Caisse de Liquidation des Affaires en Marchandises; 1983, Kuala Lumpur Commodity Clearing House; 1987, Hong Kong Futures Exchange.
  - ⟶ Close-to-default CCPs: 1987, CME and OCC, USA; 1999, BM&F, Brazil.
- Anyway, there is undoubtedly an important exposure netting benefit with CCPs, see Kiff (2009).
  - ⟶ Yet, disomogenous asset classes or geographical areas may reduce netting efficiency, see Cont and Kokholm (2013) in contrast with Duffie and Zhu (2011).

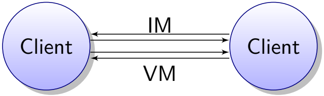
Centralized Clearing – IV



Bilateral trades and exposures without CCPs (on the left) and with CPPs (on the right). Each node lists the sum of positive exposures, each arrow the due cash flows. The diagram refers to the discussion in Kiff (2009).

Bilateral Contracts – I

- If one decides not to trade through a CCP, one may still decide to exchange collateral margins daily with the counterparty in a more private setting.
- In 2011 ISDA developed a proposal for CSA agreements which is in accordance with the collateralization practices adopted by clearing houses, and then issued in 2013 as standard CSA (SCSA).
  - ⟶ The SCSA aims at a similar treatment of collateralization for bilateral and cleared trades.
  - ⟶ It restricts eligible collateral for variation margins to cash.
  - ⟶ It promotes the adoption of overnight rate as collateral rate removing currency options.



Bilateral Contracts – II

- Different prescriptions on variation and initial margins re-hypothecation rules affects funding costs.
- In February 2013 the Basel Committee on Banking Supervision (BCBS) and the International Organization of Securities Commissions (IOSCO) issued a second consultative document on margin requirements for non-centrally cleared derivatives.
  - $\longrightarrow$ The aim is to introduce minimum standards for initial margin posting for non-centrally cleared derivatives.
- The document discusses the methodologies for calculating initial and variation margins in OTC derivatives traded between financial firms and systemically-important non-financial entities.
- The principles guiding the proposal promote a margining practice similar to the one adopted for centrally cleared products.

Variation and Initial Margins – I

- We introduce one variation margin (M_t) and two initial margin accounts (N_t^C and N_t^I).
  - $\longrightarrow$ Each party of the deal has is own initial margin accounts.
  - $\longrightarrow$ In a CCP cleared contract only the client is posting the initial margin ($N_t^I = 0$).

$$C_t := M_t + N_t^C + N_t^I \,, \quad N_t^C \geq 0 \,, \quad N_t^I \leq 0$$

  where all cash flows are from the point of view of the investor.
- In case of an hedging strategy implemented by means of collateralized instruments, we should consider also their collateral accounts.
  - $\longrightarrow$ We assume for the discussion that hedging instruments are collateralized only by posting the variation margin.

$$C_t^H := M_t^H$$

Variation and Initial Margins – II

- We assume that only the variation margin can be re-hypothecated, so that we can use it to fund the hedge.
- Thus, we can replicate the derivative in term of cash and risky assets as given by

$$V_t = F_t + M_t + H_t - M_t^H \tag{12}$$

  where V_t is the derivative price, while F_t and H_t are respectively the cash and risky part of the replica.
- Notice that, since the hedging portfolio must offset the risky part of the replica, we have that

  $\longrightarrow$ $-H_t$ is the hedging portfolio, and

  $\longrightarrow$ $-F_t$ is the cash needed to implement the hedging strategy.

Derivative Cash Flows – I

- In order to price a financial product (for example a derivative contract), we have to consider all the cash flows occurring after the trading position is entered. We can group them as follows:
  - $\longrightarrow$ product cash flows (e.g. coupons, dividends, premiums, final payout, etc. . .) inclusive of hedging instruments cash flows;
  - $\longrightarrow$ cash flows required by the collateral margining procedure;
  - $\longrightarrow$ cash flows required by the funding and hedging procedures;
  - $\longrightarrow$ cash flows occurring upon default (close-out procedure).
- In order to model these additional terms, we follow Pallavicini, Perini, Brigo (2011) which consider them as additional coupons (or dividends).
  - $\longrightarrow$ As a consequence derivative contracts, even if they do not pay coupons, behave as assets paying dividends.

Derivative Cash Flows – II

- We start from derivative cash flows ϕ_{T_i}, and we define the cumulated coupon process π_t as

$$\pi_t := \sum_{i=1}^{n} 1_{\{t > T_i\}} \phi_{T_i}$$

- Thus, the first contribution to the derivative price is given by leading to

$$V_t := \mathbb{E}_t[\,\Pi(t, T \wedge \tau) + \ldots\,]$$

$\longrightarrow$ $\tau := \tau_C \wedge \tau_I$ is the first default time, and
$\longrightarrow$ $\Pi(t,u)$ is the sum of all discounted payoff terms up to time u,

where

$$\Pi(t,T) := \int_t^T d\pi_u \, D(t,u)$$

- We calculate prices by discounting cash-flows under risk-neutral measure by following Pallavicini, Perini and Brigo (2011,2012).

Collateral Procedure

- As second contribution we consider the collateralization procedure, and we add its cash flows.

$$V_t := \mathbb{E}_t\big[\,\Pi(t, T\wedge\tau) + \gamma(t, T\wedge\tau; C, C^H) + \ldots\,\big]$$

where $\gamma(t, u; C, C^H)$ is the collateral margining costs up to time u.

- The margining costs can be deduced by the summing all the cash flows required by the collateral procedure.
- They can be expressed as the cost-of-carry of the collateral accounts. Thus, we can write

$$\gamma(t, T; C, C^H) := \int_t^T du\, D(t,u)\,\big((r_u - c_u)C_u - (r_u - h_u)C^H_u\big)$$

where c_t and h_t are respectively the collateral (or repo) rates of the two accounts C_t and C^H_t.

Funding Costs – I

- As third contribution we consider the costs of funding the hedge, and we add its cash flows.

$$\begin{aligned} V_t &:= \mathbb{E}_t\big[\,\Pi(t, T\wedge\tau) + \gamma(t, T\wedge\tau; C, C^H_t)\,\big] \\ &+ \mathbb{E}_t\big[\,\varphi(t, T\wedge\tau; F) + \ldots\big] \end{aligned}$$

  where $\varphi(t, u; F)$ is the cost of funding the hedge up to time u.

- The cash flows due to the funding costs are equal to the cost-of-carry of the funding account

$$\varphi(t, u; F) := \int_t^T du\,(r_u - f_u) F_u D(t, u)$$

  where the borrowing and lending rates f_t are given by the Treasury.

Funding Costs – II

- Additional terms can be added if the collateral account is segregated to take into account its funding costs.
  - $\longrightarrow$ Here, we assume that the variation margin may be re-hypothecated, while the initial margin is segregated.

$$
\begin{aligned}
\varphi(t,u;F) \;:=\; & \int_t^T du\,(r_u - f_u) F_u D(t,u) \\
- & \int_t^T du\,\left((r_u - f_u^{N^C}) N_u^C + (r_u - f_u^{N^I}) N_u^I \right) D(t,u)
\end{aligned}
$$

where the borrowing and lending rates $f_t^{N^C}$ and $f_t^{N^I}$ are given by the Treasury.

- These rates are possibly different from f_t if the segregated accounts participate in different netting sets.

Close-Out Netting Rules

- As fourth and last contribution we consider the cash flow occurring on default event, and we have

$$\begin{aligned} V_t &:= \mathbb{E}_t[\,\Pi(t, T\wedge\tau) + \gamma(t, T\wedge\tau; C) + \varphi(t, T\wedge\tau; F)\,] \\ &+ \mathbb{E}_t\left[\,1_{\{t<\tau<T\}} D(t,\tau)\theta_\tau(C,\varepsilon)\,\right] \end{aligned}$$

  $\longrightarrow$ $\theta_\tau(C,\varepsilon)$ is the on-default cash flow inclusive of CVA and DVA, and
  $\longrightarrow$ ε_τ is the amount of losses or costs the surviving party would incur on default event (close-out amount).

- The cash flow depends on the value of the collateral account and on the residual value of the claim being traded at default (close-out amount).
- The above pricing formula is described in detail in Pallavicini, Perini and Brigo (2011,2012).

  $\longrightarrow$ Initial margins are then considered in Brigo and Pallavicini (2014).

Pricing Master Formula – I

- If we add all the contributions we obtain

$$
\begin{aligned}
V_t &= \int_t^T \mathbb{E}\big[\, 1_{\{u<\tau\}} D(t,u)(d\pi_u + 1_{\{\tau\in du\}}\theta_u) \,|\, \mathcal{G}_t \,\big] \\
&+ \int_t^T du\, \mathbb{E}\big[\, 1_{\{u<\tau\}} D(t,u) \left((f_u - c_u) M_u - (f_u - h_u) M_u^H \right) \,|\, \mathcal{G}_t \,\big] \\
&+ \int_t^T du\, \mathbb{E}\Big[\, 1_{\{u<\tau\}} D(t,u) \Big((f_u^{N^C} - c_u) N_u^C + (f_u^{N^I} - c_u) N_u^I \Big) \,|\, \mathcal{G}_t \,\Big] \\
&+ \int_t^T du\, \mathbb{E}\big[\, 1_{\{u<\tau\}} D(t,u)(r_u - f_u)(V_u - H_u) \,|\, \mathcal{G}_t \,\big]
\end{aligned}
$$

- The above formula is an implicit calculation for V_t, since it appears also on the right-hand side.

  $\longrightarrow$ An equivalent BSDE formulation can be found in Crépey (2011) or Pallavicini, Perini and Brigo (2011).

Pricing Master Formula – II

- The hedging account and its collateral assets can be described by three relevant cases.
  - $\longrightarrow$ Hedging on the spot market: $h_t \doteq f_t$.
  - $\longrightarrow$ Hedging on the repo market: h_t is the repo rate, $M_t^H \doteq H_t$.
  - $\longrightarrow$ Hedging via perfectly collateralized assets: $h_t \doteq c_t$, $M_t^H \doteq H_t$.
- These three cases can be summarized as

$$\begin{aligned} V_t &= \int_t^T \mathbb{E}\left[1_{\{u<\tau\}} D(t,u)(d\pi_u + 1_{\{\tau\in du\}}\theta_u + (f_u - c_u)M_u\,du) \,|\, \mathcal{G}_t \right] \\ &+ \int_t^T du\,\mathbb{E}\left[1_{\{u<\tau\}} D(t,u) \left((f_u^{N^C} - c_u)N_u^C + (f_u^{N^I} - c_u)N_u^I \right) \,|\, \mathcal{G}_t \right] \\ &+ \int_t^T du\,\mathbb{E}\left[1_{\{u<\tau\}} D(t,u) \left((r_u - f_u)V_u - (r_u - h_u)H_u \right) \,|\, \mathcal{G}_t \right] \end{aligned}$$

Pricing Master Formula – III

Pricing Master Formula – Brigo and Pallavicini (2014)

The price of a derivative inclusive of counterparty risk and funding costs does not depend on the risk-free rate, and it is given by

$$\begin{aligned} V_t &= \int_t^T \mathbb{E}^h\Big[\, 1_{\{u<\tau\}} D(t,u;f)(d\pi_u + 1_{\{\tau\in du\}}\theta_u) \mid \mathcal{G}_t \Big] \\ &+ \int_t^T du\, \mathbb{E}^h\Big[\, 1_{\{u<\tau\}} D(t,u;f)(f_u - c_u) M_u \mid \mathcal{G}_t \Big] \\ &+ \int_t^T du\, \mathbb{E}^h\Big[\, 1_{\{u<\tau\}} D(t,u;f)\Big((f_u^{N^C} - c_u) N_u^C + (f_u^{N^I} - c_u) N_u^I\Big) \mid \mathcal{G}_t \Big] \end{aligned} \tag{13}$$

where the expectation is taken under a measure $\mathbb{Q}^h$ under which the underlying risk factors grow at rate h_t.

Talk Outline

Payoff Risk – I

- We are able to write the payoff to be evaluated when taking into account counterparty risk in all its details?
  - ⟶ In a discussion panel at a dedicated conference on counterparty risk, it has been said that, asking to five banks the way they compute CVA, the result was collecting fifteen different ways depending on functions and deals.
- Here, we choose a possible definition for the collateral accounts and the close-out amount processes.
  - ⟶ The collateral procedure is defined by the CSA, but the way to calculate the collateral account is to be agreed by the counterparties.
  - ⟶ The close-out amount is loosely defined by ISDA as a replacement cost.
- In this section we make a first rough approximation which will be improved in the next section, where we will discuss the close-out procedure in details.

Payoff Risk – II

- As a first step we define the on-default cash flow.
  - $\longrightarrow$ We follow Brigo, Capponi, Pallavicini and Papatheodorou (2011).
- We assume that
  - $\longrightarrow$ the variation margin M_t can be re-hypothecated, and
  - $\longrightarrow$ initial margins are not required, namely $N_t^C \doteq N_t^I \doteq 0$.
- In such settings we have

$$\begin{aligned} \theta_\tau &\doteq \varepsilon_\tau \\ &- 1_{\{\tau=\tau_C<\tau_I\}} \mathrm{L}_{\mathrm{GD}_C} (\varepsilon_\tau - M_{\tau^-})^+ \\ &- 1_{\{\tau=\tau_I<\tau_C\}} \mathrm{L}_{\mathrm{GD}_I} (\varepsilon_\tau - M_{\tau^-})^- \end{aligned} \quad (14)$$

where loss-given-defaults are defined as $\mathrm{L}_{\mathrm{GD}_C} := 1 - R_C$, and so on.

Effective Discount Approximation – I

- We extend Pallavicini and Brigo (2013) to specialize the master pricing equation by choosing the collateral process and the close-out amount as given by

$$M_t \doteq \alpha_t V_t \;, \quad \varepsilon_\tau \doteq \beta_\tau V_{\tau^-} \tag{15}$$

  where $\alpha_t \geq 0$ is the collateral fraction, and β_τ the devaluation factor.

- We have some special cases:
  1. no collateralization: $\alpha_t = 0$, e.g. IRS with a corporate;
  2. partial collateralization: $0 < \alpha_t < 1$, e.g. IRS with asymmetric CSA;
  3. perfect collateralization: $\alpha_t = 1$, e.g. standard IRS;
  4. over-collateralization: $\alpha_t > 1$, e.g. IRS with haircuts.

- When $\beta_\tau = 1$ with have not gap risk.

Effective Discount Approximation – II

- We obtain after some algebra in case of $\mathcal{F}$-conditional independence between the default times

$$V_t = 1_{\{\tau > t\}} \int_t^T \mathbb{E}^h[\, D(t,u; f + \xi)\, d\pi_u \mid \mathcal{F}_t \,] \tag{16}$$

where we define the spread ξ_t over funding as

$$\begin{aligned}
\xi_t \quad := & \quad -\alpha_t(f_t - c_t) + (\lambda_t^I + \lambda_t^C)(1 - \beta_t) \\
+ & \quad (\beta_t - \alpha_t)^+ \left(\lambda_t^C \mathrm{L}_{\mathrm{GD}C} 1_{\{V_t > 0\}} + \lambda_t^I \mathrm{L}_{\mathrm{GD}I} 1_{\{V_t < 0\}}\right) \\
+ & \quad (\beta_t - \alpha_t)^- \left(\lambda_t^I \mathrm{L}_{\mathrm{GD}I} 1_{\{V_t > 0\}} + \lambda_t^C \mathrm{L}_{\mathrm{GD}C} 1_{\{V_t < 0\}}\right)
\end{aligned}$$

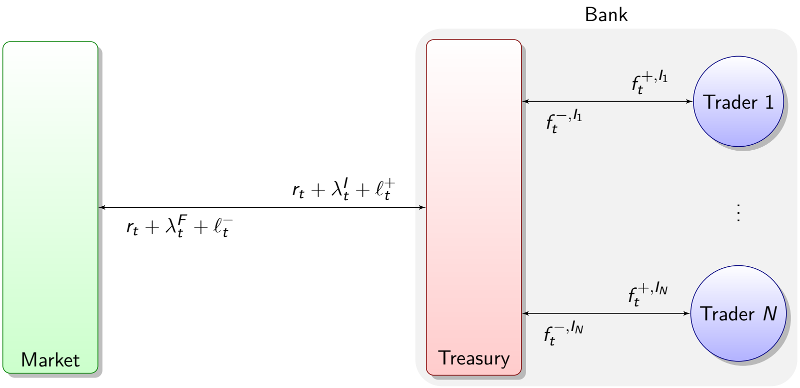
and the pre-default intensities are defined as

$$\lambda_t^I \, dt := \mathbb{Q}^c\{\, \tau_I \in dt \mid \tau_I > t, \mathcal{F}_t \,\} \;, \quad \lambda_t^C \, dt := \mathbb{Q}^c\{\, \tau_C \in dt \mid \tau_C > t, \mathcal{F}_t \,\}$$

Treasury Funding Operations – I

- The next step is describing how the Treasury department defines the funding costs entering pricing equation.
- The role of the Treasury is to manage the cash funding and investing operations, by implementing the business policy of the Bank.
  - ⟶ The Treasury is a "performance" center separated from internal business.
  - ⟶ See, for instance, Kratky and Choudhry (2012).
- The Treasury department determine the funding rate f_t for each trading desk.
  - ⟶ The funding rate may be the same for all the trading desks, or different according to the characteristics of their portfolios.
  - ⟶ The rate used to borrow cash (funding rate) may be different from the rate used to lend (investing rate).

Treasury Funding Operations – II



Treasury Funding Operations – III

- In order to select the funding rate f_t the Treasury department considers that
  - ⟶ trading positions may be netted before funding on the market;
  - ⟶ a Funds Transfer Pricing (FTP) process may be implemented to gauge the performances of different business units;
  - ⟶ a maturity transformation rule can be used to link portfolios to effective maturity dates;
  - ⟶ many source of funding can be mixed.
- Term structures of funding rates are published by the Treasury department each day.
  - ⟶ Fair-value policies may prescribe specific rules to use the term structure for some portfolios.
  - ⟶ For instance, rules for callable products could force funding on the first exercise date.

Treasury Funding Operations – IV

- Pricing with funding cost require a dynamics for funding rates, but it is very difficult to forecast the future strategies followed by the Treasury.
  - $\longrightarrow$ The term structure of funding rates is model-depend.
  - $\longrightarrow$ The option market (e.g. contingent funding derivatives) is missing.
- A tempting possibility is using the LIBOR rates as a proxy of funding rates.
  - $\longrightarrow$ This choice is widely spread, but it is very problematic, since it implies that the funding policies of the Treasury department is based on inter-bank deposits (not to speak of possible frauds in LIBOR published rates).
  - $\longrightarrow$ After the crisis only a small part of funding comes from this source.
- More likely the source of funding is the collateral portfolio, which is mainly driven by the credit spreads of the underlying names.

Modelling the Funding Rates – I

- Here, we consider the point of view of the trading desk.
- We wish to select a model for funding rates which can be used to calculate prices on the trading book.
  - $\longrightarrow$ We are not proposing a full-featured model for the Treasury liquidity policy, but only a simplified, but realistic, model to size the impact of market and credit risks in funding costs.
- We define the rate f_t by differentiating between funding and investing rates.

$$f_t := 1_{\{F_t>0\}} f_t^+ + 1_{\{F_t<0\}} f_t^- \ , \quad F_t := \sum_i F_t^i \tag{17}$$

  where the summations are over the funding netting set, and F_t^i is the cash needed to implement the hedging strategy of each set.
- If the netting set is the sum of all the transaction of the Bank, we could obtain that $\sum_i F_t^i > 0$ at any time.
  - $\longrightarrow$ A possible proxy for funding costs can be the CDS/Bond basis.

Modelling the Funding Rates – II

- By following Pallavicini and Brigo (2013), we consider the following proxy for funding rates

$$f_t^- \doteq e_t + w^-(t) + w^P(t)\lambda_t^P$$

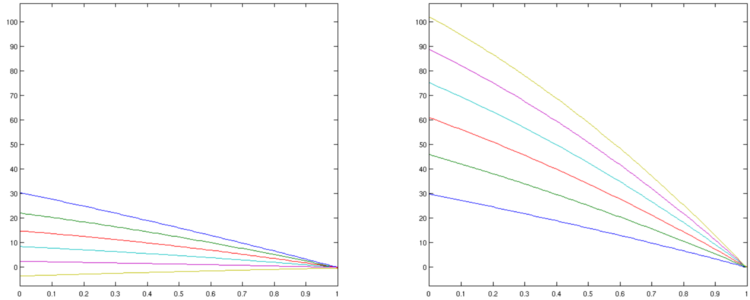
and

$$f_t^+ \doteq e_t + w^+(t) + w^P(t)\lambda_t^P + w^I(t)\lambda_t^I$$

where e_t is the overnight rate, λ_t^P is the typical default intensity of the names of the collateral portfolio, and λ_t^I is the default intensity of the investor.

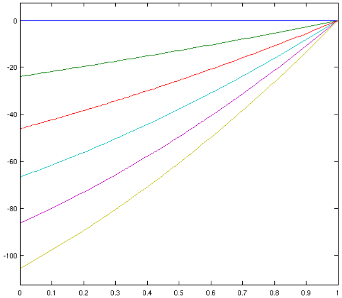
- The w's can be calibrated to Treasury data, since they represent the Treasury liquidity policy.
  - ⟶ This choice is enough flexible to describe many different policies.
  - ⟶ By setting $w^+(t) = w^-(t)$ and $w^I(t) = 0$ we obtain the same rates for funding and investing.

Funding Costs and Partial Collateralization



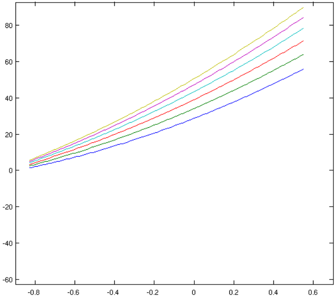
Price for a receiver IRS (left) and for shorting a payer IRS (right) vs. collateral fraction α for different borrowing rates, while keeping the lending rate equal to the overnight rate. Investor and counterparty with high-risk settings. $w^I \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$, $w^P = 0$, $w^{\pm} = 0$. See Pallavicini and Brigo (2013) for details.

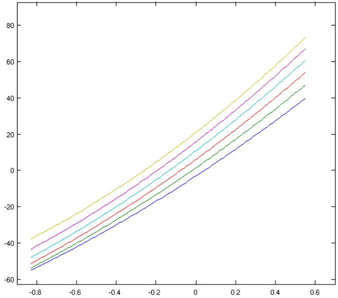
Bid-Ask Spreads and Partial Collateralization



Bid-ask spread for an IRS vs. collateral fraction α for different borrowing rates, while keeping the lending rate equal to the overnight rate. Investor and counterparty with high-risk settings. $w^I \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$, $w^P = 0$, $w^{\pm} = 0$. See Pallavicini and Brigo (2013) for details.

Funding Costs and Wrong-Way Risk





Price for a receiver IRS vs. correlation between credit-spreads and overnight rate for different funding rates. Collateralization is off ($\alpha = 0$). Investor and counterparty with high-risk settings. Collateral portfolio with mid-risk settings. Left: $w^I = 0$, $w^P \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$, $w^{\pm} = 0$. Right: $w^I = 1$, $w^P \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$, $w^{\pm} = 0$. See Pallavicini and Brigo (2013) for details.

Margin Period of Risk – I

- The effective discount approximation assumes that
  - ⟶ variation margins and close-out amounts are proportional to the derivative price,
  - ⟶ variation margis can be re-hypothecated,
  - ⟶ initial margins are not present (we can simulate its effetcs with haircuts on variation margin),
  - ⟶ gap risk is not modelled (we can simulate its effects with haircuts on close-out amount),
  - ⟶ the default procedure happens instantaneously on the default event.
- We now continue by improving the above approximations.
- In particular, we start with a detailed descriptions of what happens on the default event to introduce a more elaborated defintion of close-out amount, on-default cash flow, and gap risk.

Margin Period of Risk – II

- The default procedures may take δ days to be completed (margin period of risk).
  - $\longrightarrow$ In such time-frame the mark-to-market value of the derivative may change considerably.
  - $\longrightarrow$ Furthermore, we should consider the possibility that also the surviving party may default in this lapse of time.
- On default event the survivng party evaluates the exposure ε, also known as close-out amount.
  - $\longrightarrow$ This valuation is defined by ISDA documentation as a replacement deal on the market.
  - $\longrightarrow$ The surviving party may take into account the costs of terminating, liquidating or re-establishing any hedge or related trading position and, furthermore, can consider the cost of funding.
  - $\longrightarrow$ See Brigo, Capponi, Pallavicini and Papatheodorou (2011) or Durand and Rutkowski (2013) for details.

Margin Period of Risk – III

- By following Brigo and Pallavicini (2014) we can model the on-default cash flow θ as given by

$$\begin{aligned} \theta_\tau &:= \mathbb{E}^h\left[D(\tau,\tau+\delta;f^S)\vartheta_{\tau+\delta}(\varepsilon;M,N^C,N^I) \,|\, \mathcal{G}_\tau \right] \\ &\approx \mathbb{E}^h\left[\vartheta_{\tau+\delta}(\varepsilon;M,N^C,N^I) \,|\, \mathcal{G}_\tau \right] \end{aligned} \tag{18}$$

  where f_t^S is the funding rate of the surviving party.
- Collateral accounts M, N^C and N^I impact the valuation of ϑ with their values just before the default event, namely at τ^-.
- The exposure ε impacts the valuation of ϑ with its value at the completion of the default procedure, namely at $\tau+\delta$.
- In case of further default of the surviving party, this is substituted by his bankruptcy trustee in the default procedure.
  - $\longrightarrow$ Thus, the cash flows originating from such party are reduced by a recovery rate.

Close-Out Netting Rules – I

- We can now investigate the close-out netting rules to calculate $\vartheta_{\tau+\delta}$ in terms of the close-out amount and collateralization accounts.
  - $\longrightarrow$ We have different situations according to the sign of the close-out amount and the collateral accounts.
- Close-out netting rules decribe what happens on default event concerning
  - $\longrightarrow$ the collateral accounts at disposal to reduce the exposure (otherwise losses at L_{GD_C} or L_{GD_I} level),
  - $\longrightarrow$ claiming assets posted as collateral after netting occurs (otherwise losses at $\text{L}_{\text{GD}'_C}$ or $\text{L}_{\text{GD}'_I}$ level),
  - $\longrightarrow$ withdrawing assets received as collateral after netting occurs.
- Moreover, the rules must differentiate among re-hypothecable assets and assets stored in segregated accounts.
- Once the cash flows exchanged on $\tau + \delta$ are known, we can evaluate $\vartheta_{\tau+\delta}$, and in turn θ_τ.

Close-Out Netting Rules – II

Close-Out Netting Rules – Brigo and Pallavicini (2014)

The on-default cash flow according to ISDA close-out netting rules is

$$
\begin{aligned}
\theta_\tau &= \mathbb{E}^h[\,\varepsilon_{\tau+\delta}\,|\,\mathcal{G}_\tau\,] \\
&- \mathbb{E}^h\big[\,1_{\{\tau_C<\tau_I+\delta\}}\textsc{Lgd}_C((\varepsilon_{\tau+\delta}-N^C_{\tau-})^+ - M^+_{\tau-})^+\,|\,\mathcal{G}_\tau\,\big] \\
&- \mathbb{E}^h\big[\,1_{\{\tau_C<\tau_I+\delta\}}\textsc{Lgd}'_C((\varepsilon_{\tau+\delta}-N^C_{\tau-})^- - M^-_{\tau-})^+\,|\,\mathcal{G}_\tau\,\big] \\
&- \mathbb{E}^h\big[\,1_{\{\tau_I<\tau_C+\delta\}}\textsc{Lgd}_I((\varepsilon_{\tau+\delta}-N^I_{\tau-})^- - M^-_{\tau-})^-\,|\,\mathcal{G}_\tau\,\big] \\
&- \mathbb{E}^h\big[\,1_{\{\tau_I<\tau_C+\delta\}}\textsc{Lgd}'_I((\varepsilon_{\tau+\delta}-N^I_{\tau-})^+ - M^+_{\tau-})^-\,|\,\mathcal{G}_\tau\,\big]
\end{aligned}
\tag{19}
$$

- The first term is the replacement price of the deal.
- The second and third terms are the counterparty risk due to the counterparty default (CVA).
- The fourth and fifth terms are the counterparty risk due to the investor default (DVA).

Close-Out Amount Evaluation – I

- We focus now on the evaluation of the the close-out amount ε entering the equation of the on-default cash flow.
- ISDA documentation prescribes to calculate close-out amounts as replacement prices.
  - $\longrightarrow$ Yet, we do not know the counterparty of the replacement deal, and, as a consequence, we cannot seize his credit charge into the pricing equations in a precise way.
- Here, we assume that the close-out amount is equal to the mark-to-market of the derivative contract considered between two default-free counterparties in case of perfect collateralization.

$$\varepsilon_{\tau+\delta}(\tau, T) := \int_{\tau}^{T} \mathbb{E}^h[\, D(\tau, u; c) d\pi_u \,|\, \mathcal{G}_{\tau+\delta}\,] \tag{20}$$

Close-Out Amount Evaluation – II

- If we assume that the variation margin can be re-hypothecated, so that $\text{L}\textsc{gd}'_C = \text{L}\textsc{gd}_C$ and $\text{L}\textsc{gd}'_I = \text{L}\textsc{gd}_I$, we obtain by direct substitution

$$
\begin{aligned}
V_t &= \varepsilon_t(t,T) \\
&+ \int_t^T du\, \mathbb{E}^h\Big[\, 1_{\{u<\tau\}} D(t,u;f)((f_u^{N^C} - c_u)N_u^C + (f_u^{N^I} - c_u)N_u^I) \mid \mathcal{G}_t \Big] \\
&- \int_t^T \mathbb{E}^h\Big[\, 1_{\{\tau\in du\}} 1_{\{\tau_C<\tau_I+\delta\}} \text{L}\textsc{gd}_C D(t,u;f) \left(\Delta^C_{u+\delta}\right)^+ \mid \mathcal{G}_t \Big] \\
&- \int_t^T \mathbb{E}^h\Big[\, 1_{\{\tau\in du\}} 1_{\{\tau_I<\tau_C+\delta\}} \text{L}\textsc{gd}_I D(t,u;f) \left(\Delta^I_{u+\delta}\right)^- \mid \mathcal{G}_t \Big]
\end{aligned}
$$

where we define the gap risk Δ as given by

$$
\Delta^C_{\tau+\delta} := \varepsilon_{\tau+\delta}(\tau,T) - N^C_{\tau^-} - M_{\tau^-} \;, \quad \Delta^I_{\tau+\delta} := \varepsilon_{\tau+\delta}(\tau,T) - N^I_{\tau^-} - M_{\tau^-} \;.
$$

Gap Risk – I

- Gap risk may be almost instantaneous (default contagion), or may build during the margin period of risk (mark-to-market volatility).
  $\longrightarrow$ The initial margins requested by a CCP protect from such risk.
- The relevance of gap risk depends on the asset class we are considering.
  $\longrightarrow$ CDS prices are heavily affected by instantaneous gap risk, since the mark-to-market of a CDS jumps when one of the counterparties defaults.
- We can highlight the different contributions hinted above by rewriting the gap risk definition in the following form

$$\Delta_{\tau+\delta} = \Delta_{\tau^-}^{\text{Mismatch}} + \Delta_{\tau}^{\text{Contagion}} + \Delta_{\tau+\delta}^{\text{MtM}} \tag{21}$$

- We are able by a suitable choice of the collateral accounts only to drop the mismatch term of gap risk, since the other two terms are fixed after the default event happens.

Gap Risk – II

- The component of the gap risk due to a mismatch between the variation margin account and the value of the close-out amount is given by

$$\Delta_{\tau^-}^{\text{Mismatch}} := \varepsilon_{\tau^-}(\tau, T) - M_{\tau^-}$$

- The component of the gap risk due to an instantaneous contagion effect at default time is given by

$$\Delta_{\tau}^{\text{Contagion}} := \varepsilon_{\tau}(\tau, T) - \varepsilon_{\tau^-}(\tau, T) - N_{\tau^-}^{\text{Contagion}}$$

where we split the initial margin contribution into two parts:

$$N_{\tau^-} := N_{\tau^-}^{\text{Contagion}} + N_{\tau^-}^{\text{MtM}} .$$

- The component of the gap risk due to a movement in the mark-to-market of the close-out amount between the default event and the end of the default procedure is given by

$$\Delta_{\tau+\delta}^{\text{MtM}} := \varepsilon_{\tau+\delta}(\tau, T) - \varepsilon_{\tau}(\tau, T) - N_{\tau^-}^{\text{MtM}}$$

Variation and Initial Margin Estimates – I

- The variation margin in bilateral contracts can be defined to include all the derivative cash flows with the exception of those depending on the funding costs of the counterparties.
- In the CCP case we usually find contractual rules explaining how to calculate the variation margin by discounting the derivative coupons at an official rate issued by the CCP at the end of each trading day.
- Thus, in general, we can assume

$$M_t \doteq \alpha_t \, \varepsilon_t(t, T) \tag{22}$$

  where the collateral fraction α_t will usually be equal to 1.
- If $\alpha_t \doteq 1$ we can drop the mismatch gap-risk term, namely we get

$$\alpha_t \doteq 1 \Longrightarrow \Delta_{\tau^-}^{\text{Mismatch},C} = \Delta_{\tau^-}^{\text{Mismatch},I} = 0$$

Variation and Initial Margin Estimates – II

- Initial margins are strongly dependent on the particular asset class of the derivative.
- LCH, a CCP clearing IRS contracts, consider a single source of risk to estimate initial margins.
  - $\longrightarrow$ Interest-rate uncertainty, analyzed in term of a historical metric.
- ICE, a CCP clearing CDS contracts, considers seven(!) different sources of risk to estimate initial margins.
  - $\longrightarrow$ Credit-spread, interest-rate and recovery-rate uncertainties.
  - $\longrightarrow$ Jump risk, namely default contagion effects.
  - $\longrightarrow$ Basis risk, namely mismatches between particular contracts and market proxies.
  - $\longrightarrow$ Liquidity risk, by observing bid/ask spreads and via price discovery.
  - $\longrightarrow$ Concentration risk, namely systemic risk associated with large portfolios.

Variation and Initial Margin Estimates – III

- Here, we focus on interest-rate derivatives.
  - $\longrightarrow$ We disregard the contagion component of gap risk, and
  - $\longrightarrow$ by following LCH we consider only the mark-to-market component of gap risk
- Thus, we assume

$$N_{\tau^-}^{\text{Contagion},C} \doteq N_{\tau^-}^{\text{Contagion},I} \doteq 0 \tag{23}$$

leading to

$$\Delta_{\tau}^{\text{Contagion},C} \doteq \Delta_{\tau}^{\text{Contagion},I} \doteq 0$$

- The gap risk arising from the mark-to-market term is usually analyzed in terms of historical Value-at-Risk (VaR) or Expected Shortfall (ES) estimates.

Variation and Initial Margin Estimates – IV

- We estimate the initial margin posted to protect from mark-to-market movements as the protection against the worst movement of the contract due to market risk within δ days at a confidence level q according to VaR risk metric.
- In the bilateral and in the CCP case we have

$$N_t^{\mathrm{MtM},C} \doteq \inf \left\{ x \geq 0 : \mathbb{Q}^h \{ \varepsilon_{t+\delta}(t,T) - \varepsilon_t(t,T) < x \mid \mathcal{F}_t \} > q \right\} \quad (24)$$

and only for bilateral contracts under CSA

$$N_t^{\mathrm{MtM},I} \doteq \sup \left\{ x \leq 0 : \mathbb{Q}^h \{ \varepsilon_{t+\delta}(t,T) - \varepsilon_t(t,T) > x \mid \mathcal{F}_t \} > q \right\} \quad (25)$$

where we approximate the risk metric by using the pricing measure in spite of the physical measure, since we need to insert such estimates into a pricing equation.

Switching to Market Filtration – I

- We can now conclude this section by plugging the estimates of variation and initial margins into the pricing equations.
- We model derivative funding costs as in the effective discount approximation case with $w^P_t \doteq 0$ and $w^\pm_t \doteq \ell^\pm_t$, so that

$$f^\pm_t \doteq e_t + \ell^\pm_t$$

while the funding rates for the initial margin accounts are selected as

$$f^{N^C}_t \doteq e_t + \ell^{N^C}_t \ , \quad f^{N^I}_t \doteq e_t + \ell^{N^I}_t$$

- We assume that the liquidity bases are related to the pre-default intensities of the two names λ^I_t and λ^C_t.
- Furhtermore, for interest-rate derivatives we can assume that the collateralization rate is the overnight rate, so that

$$h_t \doteq c_t \doteq e_t$$

Switching to Market Filtration – II

- By plugging in all the definitions we obtain

$$
\begin{aligned}
V_t &= \varepsilon_t(t,T) \\
&+ \int_t^T du\, \mathbb{E}^e\Big[1_{\{u<\tau\}} D(t,u;f)(\ell_u^{N^C} N_u^C + \ell_u^{N^I} N_u^I) \mid \mathcal{G}_t \Big] \\
&- \int_t^T \mathbb{E}^e\Big[1_{\{\tau\in du\}} 1_{\{\tau_C<\tau_I+\delta\}} \mathrm{LGD}_C D(t,u;f) \left(\Delta_{u+\delta}^{\mathrm{MtM},C}\right)^+ \mid \mathcal{G}_t \Big] \\
&- \int_t^T \mathbb{E}^e\Big[1_{\{\tau\in du\}} 1_{\{\tau_I<\tau_C+\delta\}} \mathrm{LGD}_I D(t,u;f) \left(\Delta_{u+\delta}^{\mathrm{MtM},I}\right)^- \mid \mathcal{G}_t \Big] .
\end{aligned}
$$

- If we assume $\mathcal{F}$-conditional independence between the default times we can switch to market filtration, since in our case also the margin accounts and the gap risks are $\mathcal{F}$-adapted processes.

Switching to Market Filtration – III

- Thus, we can write the pre-default price process as given by

$$
\begin{aligned}
\widetilde{V}_t &= \varepsilon_t(t,T) \\
&+ \int_t^T du \mathbb{E}^e\Big[D(t,u;f+\lambda)(\ell_u^{N^C} N_u^C + \ell_u^{N^I} N_u^I) \mid \mathcal{F}_t \Big] \\
&- \int_t^T du \mathbb{E}^e\big[\lambda_u^{\delta,C} \, \mathrm{LGD}_C \, D(t,u;f+\lambda)(\varepsilon_{u+\delta}(u,T) - \varepsilon_u(u,T) - N_u^C)^+ \mid \mathcal{F}_t \big] \\
&- \int_t^T du \mathbb{E}^e\big[\lambda_u^{\delta,I} \, \mathrm{LGD}_I \, D(t,u;f+\lambda)(\varepsilon_{u+\delta}(u,T) - \varepsilon_u(u,T) - N_u^I)^- \mid \mathcal{F}_t \big]
\end{aligned} \tag{26}
$$

where $\lambda_t := \lambda_t^C + \lambda_t^I$, and we define

$$
\lambda_t^{\delta,C} := \lambda_t^C + \lambda_t^I \left(1 - D(t,t+\delta;\lambda^C)\right) \, , \; \lambda_t^{\delta,I} := \lambda_t^I + \lambda_t^C \left(1 - D(t,t+\delta;\lambda^I)\right)
$$

Pricing Interest-Rate Derivatives – I

- The funding rates depend on the derivative price, since they can be different acconrding to the sign of the quantity of cash needed to implement the hedging strategy, namely

$$f_t = f_t^+ 1_{\{F_t > 0\}} + f_t^- 1_{\{F_t < 0\}}$$

where we recall that

$$V_t = F_t + M_t + H_t - M_t^H$$

- In the case of interest-rate derivatives the heding instruments are perfectly collateralized, so that we have $H_t \doteq M_t^H$, leading to

$$f_t \doteq f_t^+ 1_{\{V_t > M_t\}} + f_t^- 1_{\{V_t < M_t\}} \,. \quad (27)$$

Pricing Interest-Rate Derivatives – II

- The dependency of funding rates on the derivative price introduces a fundamental non-linearity at the level of pricing operators.
- Such non-linearity was not there at the pure credit valuation adjustment (CVA/DVA) level, where non-linearity appears only in the payout.
- Thus, the pricing equation cannot be used to explicitly evaluate the price of the contract as a straightforward expectation.
  - ⟶ This equation should rather be interpreted as the solution of a BSDE with terminal condition at contract maturity T
  - ⟶ See Ma, Protter, San Martin and Torres (2002).
- The first application of BSDE in pricing is in El Karoui, Peng, Quenez (1997).
  - ⟶ First applications to funding problems in Crépey (2011).
  - ⟶ A discrete-time formulation can be found in Pallavicini, Perini and Brigo (2011).

Pricing Interest-Rate Derivatives – III

Interest-Rate Pricing Equation – Brigo and Pallavicini (2014)

The price of an interest-rate contract can be calculated as

$$\widetilde{V}_t = \varepsilon_t(t,T) + Y_t$$

where the process Y_t (with terminal condition $Y_T = 0$) is computed backwardly on the time grid $\{t = t_0, \ldots, t_i, \ldots, t_n = T\}$

$$Y_{t_i} = \mathbb{E}^e[\, Y_{t_{i+1}} D(t_i, t_{i+1}; f+\lambda) \,|\, \mathcal{F}_{t_i}\,] + \int_{t_i}^{t_{i+1}} \mathbb{E}^e[\, D(t_i, u; f+\lambda)\, d\pi_u \,|\, \mathcal{F}_{t_i}\,]$$

and the coupon process π_t as given by

$$\begin{aligned} d\pi_t \quad &:= \quad \ell_t^{N^C} N_t^C\, dt - \lambda_t^{\delta,C}\, \mathbb{E}^e\Big[\, \mathrm{L{\scriptstyle GD}}_C(\varepsilon_{t+\delta}(t,T) - \varepsilon_t(t,T) - N_t^C)^+ \,\Big|\, \mathcal{F}_t \Big]\, dt \\ &+ \quad \ell_t^{N^I} N_t^I\, dt - \lambda_t^{\delta,I}\, \mathbb{E}^e\Big[\, \mathrm{L{\scriptstyle GD}}_I(\varepsilon_{t+\delta}(t,T) - \varepsilon_t(t,T) - N_t^I)^- \,\Big|\, \mathcal{F}_t \Big]\, dt \end{aligned}$$

Numerical Scheme for the BSDE – I

- In order to implement numerically the problem we build a Euler discretization as in Ma, et al. (2002)

$$Y_{t_i} = (1 - g_{t_i}(Y_{t_i})\Delta t_i)\,\mathbb{E}^e[\,Y_{t_{i+1}}\,|\,\mathcal{F}_{t_i}\,] + \Delta\pi_{t_i}$$

  where we make explicit the dependency on the mark-to-market value of the contract and we define the rate

$$g_{t_i}(Y_{t_i}) := e_{t_i} + \lambda_{t_i} + \ell^+_{t_i} 1_{\{Y_{t_i}>0\}} + \ell^-_{t_i} 1_{\{Y_{t_i}<0\}}$$

- Then, we solve the coupled pair of equations by a fixed-point technique, leading to the following explicit scheme

$$Y_{t_i} = (1 - g_{t_i}\,(\mathbb{E}^e[\,Y_{t_{i+1}}\,|\,\mathcal{F}_{t_i}\,])\,\Delta t_i)\,\mathbb{E}^e[\,Y_{t_{i+1}}\,|\,\mathcal{F}_{t_i}\,] + \Delta\pi_{t_i}$$

Numerical Scheme for the BSDE – II

- We can reduce the variance of the simulation by means of control variate variables based on the price of the contract without funding costs.
- We define the funding-cost-free pre-default price as given by

$$
\begin{aligned}
\widetilde{V}_t^0 \;:=\;& \varepsilon_t(t,T) \\
&+ \int_t^T du\,\mathbb{E}^e\Big[\, D(t,u;e+\lambda)(\ell_u^{N^C} N_u^C + \ell_u^{N^I} N_u^I) \,\big|\, \mathcal{F}_t \Big] \\
&- \int_t^T du\,\mathbb{E}^e\big[\, \lambda_u^{\delta,C}\,\mathrm{L{\scriptstyle GD}}_C\, D(t,u;e+\lambda)(\varepsilon_{u+\delta}(u,T) - \varepsilon_u(u,T) - N_u^C)^+ \,\big|\, \mathcal{F}_t \big] \\
&- \int_t^T du\,\mathbb{E}^e\big[\, \lambda_u^{\delta,I}\,\mathrm{L{\scriptstyle GD}}_I\, D(t,u;e+\lambda)(\varepsilon_{u+\delta}(u,T) - \varepsilon_u(u,T) - N_u^I)^- \,\big|\, \mathcal{F}_t \big]
\end{aligned}
$$

- This price process can be evaluated directly by calculating the expectation, since the right-hand side does not depend on $\widetilde{V}_t^0$.

Numerical Scheme for the BSDE – III

- If we introduce the funding-cost-free price process Y^0_t as

$$Y^0_t := \widetilde{V}^0_t - \varepsilon_t(t,T)\,,$$

we get

$$Y^0_{t_i} = (1 - (e_{t_i} + \lambda_{t_i})\Delta t_i)\,\mathbb{E}^e\left[\, Y^0_{t_{i+1}} \,|\, \mathcal{F}_{t_i} \right] + \Delta\pi_{t_i}$$

- We can solve for the coupon process π_{t_i} the above equation to obtain a numerical scheme to evaluate the funding valuation adjustment process $X_t := Y_t - Y^0_t$.

$$\begin{cases} \mathcal{X}_{t_i} := \mathbb{E}^e_{t_i}\left[\, X_{t_{i+1}} \,\right] \\ \mathcal{Y}_{t_i} := \widetilde{V}^0_{t_i} - \varepsilon_{t_i}(t_i, T) + \mathcal{X}_{t_i} \\ X_{t_i} = \mathcal{X}_{t_i} - \mathcal{Y}_{t_i}\, g_{t_i}(\mathcal{Y}_{t_i})\,\Delta t_i \end{cases} \tag{28}$$

with terminal condition $X_{t_n} = 0$.

Price Decomposition for IR Derivatives

Price Decomposition for IR Derivatives – Brigo and Pallavicini (2014)

Interest-rate derivative prices can be decomposed as given by

$$\widetilde{V}_t = \mathrm{MtM}_t + \mathrm{CVA}_t + \mathrm{DVA}_t + \mathrm{MVA}_t + \mathrm{FVA}_t$$

where

$$\mathrm{MtM}_t := \varepsilon_t(t,T)\,, \quad \mathrm{FVA}_t := X_t$$

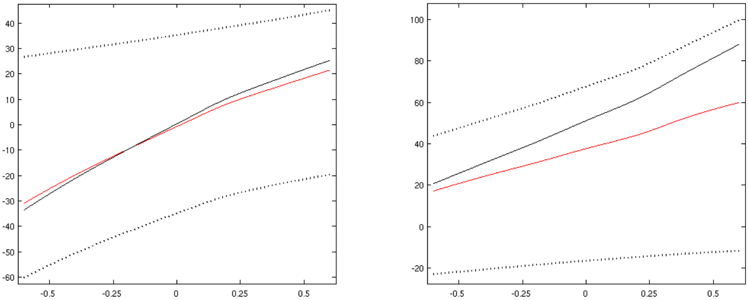
with X_t calculated by means of the numerical iterative scheme, while

$$\mathrm{CVA}_t := -\int_t^T du\,\mathbb{E}^e\big[\,\lambda_u^{\delta,C}\,\mathrm{L{\scriptstyle GD}}_C\,D(t,u;e+\lambda)(\varepsilon_{u+\delta}(u,T)-\varepsilon_u(u,T)-N_u^C)^+\mid\mathcal{F}_t\,\big]$$

$$\mathrm{DVA}_t := -\int_t^T du\,\mathbb{E}^e\big[\,\lambda_u^{\delta,I}\,\mathrm{L{\scriptstyle GD}}_I\,D(t,u;e+\lambda)(\varepsilon_{u+\delta}(u,T)-\varepsilon_u(u,T)-N_u^I)^-\mid\mathcal{F}_t\,\big]$$

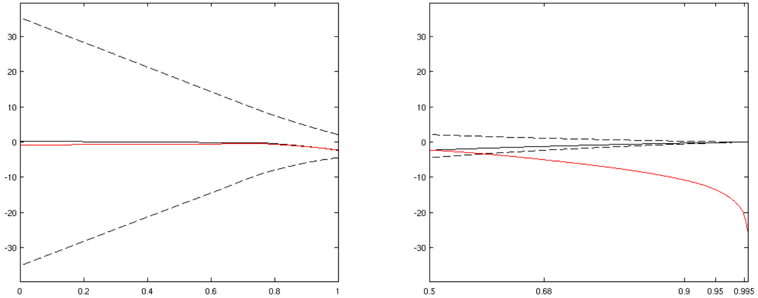
$$\mathrm{MVA}_t := \int_t^T du\,\mathbb{E}^e\Big[\,D(t,u;e+\lambda)(N_u^C\ell_u^{N^C}+N_u^I\ell_u^{N^I})\mid\mathcal{F}_t\Big]$$

Interest-Rate Swap: bilateral trades without margining



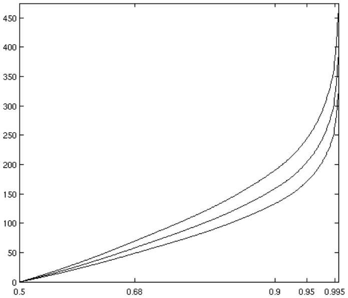
Prices of a ten-year receiver IRS, left "H/M", right "M/H". The black continuous line represents the price inclusive of CVA and DVA but not funding costs, with the dashed black lines representing separately CVA and DVA. The red continuous line is the price inclusive both of credit and funding costs. Symmetric funding policy. On the x-axis the correlation among market and credit risks.

Interest-Rate Swap: bilateral trades with margining – I



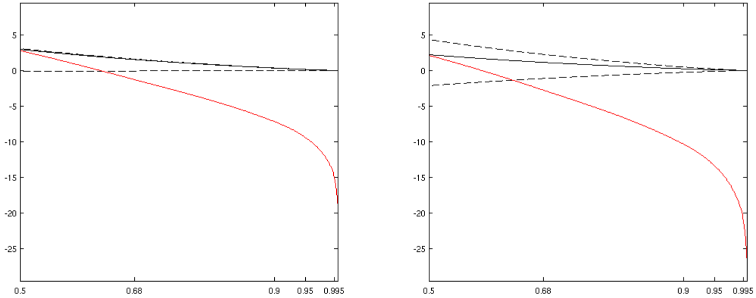
Prices of a ten-year receiver IRS, "H/M". Left: prices for different collateralization fractions α without initial margin. Right: prices with $\alpha = 1$ and initial margin posted at various confidence levels q. The black continuous line represents the price inclusive of CVA and DVA but not funding costs, with the dashed black lines representing separately CVA and DVA. The red continuous line is the price inclusive both of credit and funding costs. Symmetric funding policy.

Interest-Rate Swap: bilateral trades with margining – II



Amount of initial margin requested at contract inception for a ten-year receiver IRS, "H/M". Left: the *x*-axis lists different confidence levels, while the curves correspond to three different margin period of risks (1, 5 and 10 days). Right the *x*-axis lists different margin period of risks, while the curves correspond to three confidence levels (68%, 95% and 99.7%). Symmetric funding policy. Correlation is zero.

Interest-Rate Swap: bilateral vs. CCP trades



Prices of a ten-year receiver IRS, "M/H". Left: a CCP trade. Right: a bilateral trade. Prices with $\alpha = 1$ and initial margin posted at various confidence levels q. The black continuous line represents the price inclusive of CVA and DVA but not funding costs, with the dashed black lines representing separately CVA and DVA. The red continuous line is the price inclusive both of credit and funding costs. Symmetric funding policy. Correlation is zero.

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